Chapter 6

Moving Load and its Effects on Structural Members

Instructional Objectives:

The objectives of this lesson are as follows:

- Understand the moving load effect in simpler term
- Study various definitions of influence line
- Introduction to simple procedures for construction of influence lines

Introduction

In earlier lessons, statically determinate and statically indeterminate structural analysis was introduced under non-moving load (dead load or fixed loads). In this lecture, introduction to determination of maximum internal actions at cross-sections of members of statically determinate structured under the effects of moving loads (live loads) will be presented.

Common sense tells us that when a load moves over a structure, the deflected shape of the structural will vary. In the process, we can arrive at simple conclusion that due to moving load position on the structure, reaction values at the support will also vary.

From the designer's point of view, it is essential to have safe structure, which doesn't exceed the limits of deformations and also the limits of load carrying capacity of the structure.

Definitions of influence line

In the literature, researchers have defined influence line in many ways. Some of the definitions of influence line are given below.

- An influence line is a diagram whose ordinates, which are plotted as a function of distance along the span, give the value of an internal force, a reaction, or a displacement at a particular point in a structure as a unit load move across the structure.
- An influence line is a curve the ordinate of which at any point equals the value of some particular function due to unit load acting at that point.
- An influence line represents the variation of either the reaction, shear, moment, or deflection at a specific point in a member as a unit concentrated force moves over the member.

Construction of Influence Lines

In this section, the construction of influence lines will be discussed. Using any one of the two approaches (Figure 1), one can construct the influence line at a specific point P in a member for any parameter (Reaction, Shear or Moment)

In the present approaches it is assumed that the moving load is having dimensionless magnitude of unity. Classification of the approaches for construction of influence lines is given in Figure 1.

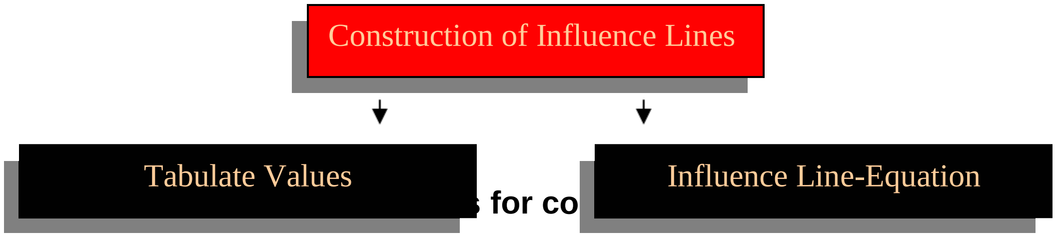


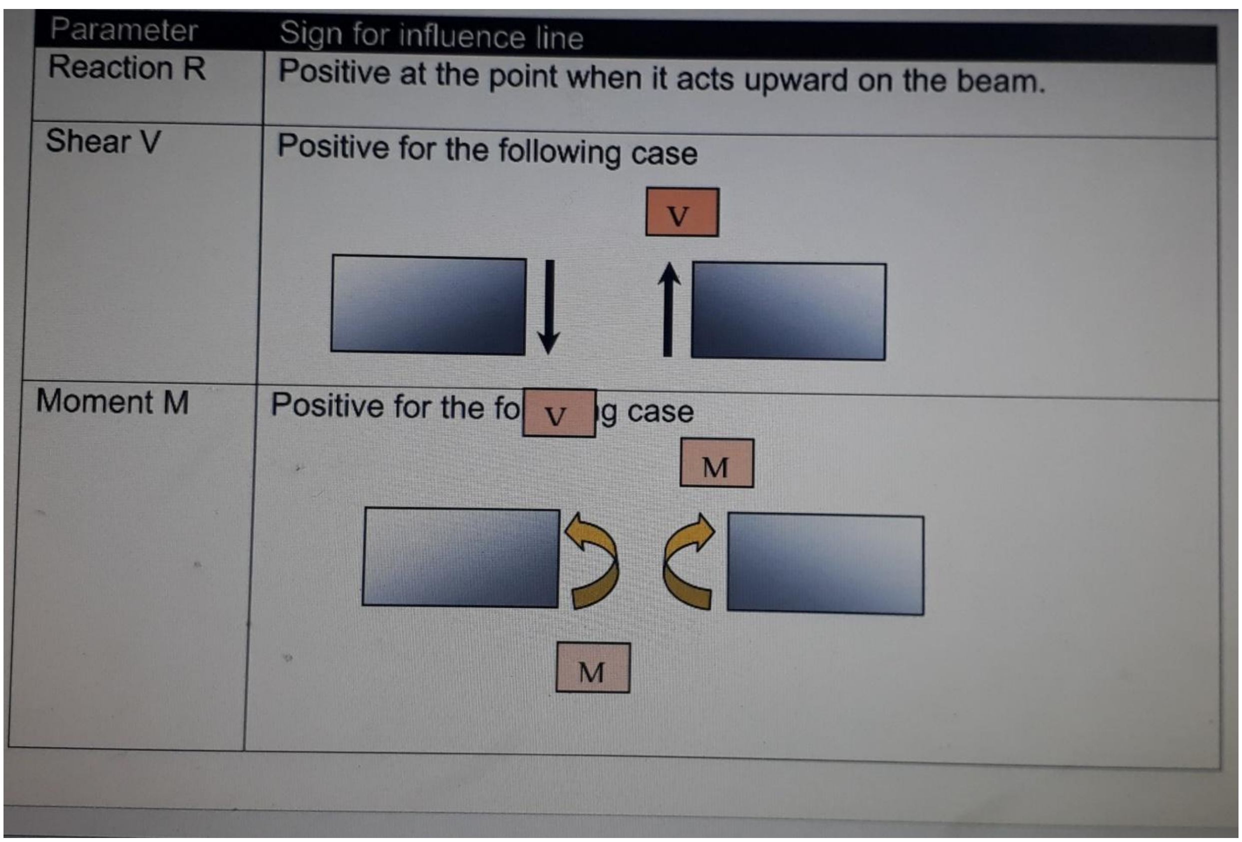
Figure 1

Tabulate Values

Apply a unit load at different locations along the member, say at ***x***. And at these locations, apply statics to compute the value of parameter (reaction, shear, or moment) at the specified point. The best way to use this approach is to prepare a table, listing unit load at x versus the corresponding value of the parameter calculated at the specific point (i.e. Reaction R, Shear V or moment M) and plot the tabulated values so that influence line segments can be constructed.

Sign Conventions

Sign convention followed for shear and moment is given below.



Parameter	Sign for influence line
Reaction R	Positive at the point when it acts upward on the beam.
Shear V	Positive for the following case
Moment M	Positive for the fo[…]g case

Influence Line Equations

Influence line can be constructed by deriving a general mathematical equation to compute parameters (e.g. reaction, shear or moment) at a specific point under the effect of moving load at a variable position x.

The above discussed both approaches are demonstrated with the help of simple numerical examples in the following paragraphs.

Numerical Examples

Example 1:

Construct the influence line for the reaction at support B for the beam of span 10 m. The beam structure is shown in Figure 2.

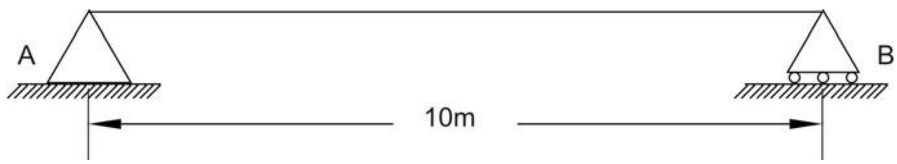


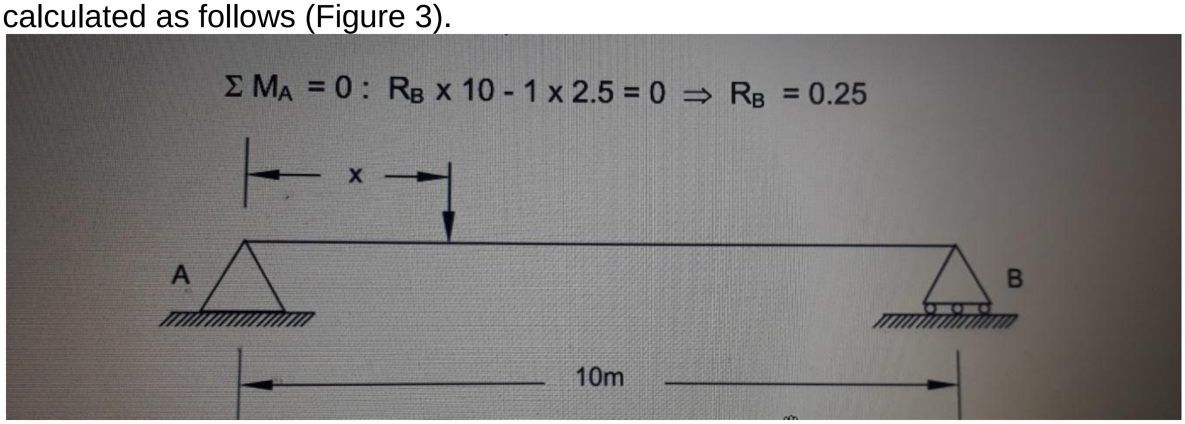
Figure 2: The beam structure

Solution:

As discussed earlier, there are two ways this problem can be solved. Both the approaches will be demonstrated here.

Tabulate values:

As shown in the figure, a unit load is places at distance x from support A and the reaction value R_B is calculated by taking moment with reference to support A. Let us say, if the load is placed at 2.5 m. from support A then the reaction R_B can be calculated as follows (Figure 3).



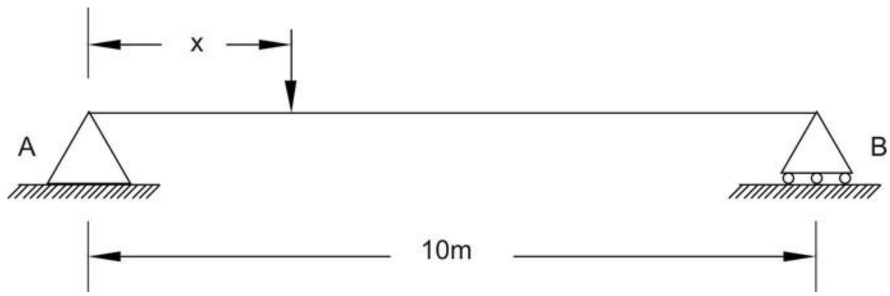


Figure 3: The beam structure with unit load

Similarly, the load can be placed at 5.0, 7.5 and 10 m. away from support A and reaction R_B can be computed and tabulated as given below.

x	R_B
0	0.0
2.5	0.25
5.0	0.5
7.5	0.75
10	1

Graphical representation of influence line for R_B is shown in Figure 4.

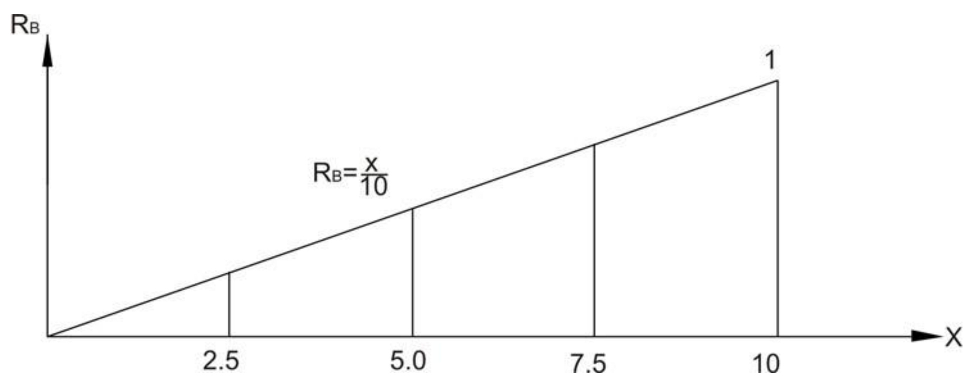


Figure 4: Influence line for reaction R_B.

Influence Line Equation:

When the unit load is placed at any location between two supports from support A at distance x then the equation for reaction R_B can be written as

$$\Sigma M_A = 0 : R_B \times 10 - x = 0 \Rightarrow R_B = x/10$$

The influence line using this equation is shown in Figure 4.

Example 2:

Construct the influence line for support reaction at B for the given beam as shown in Fig 5.

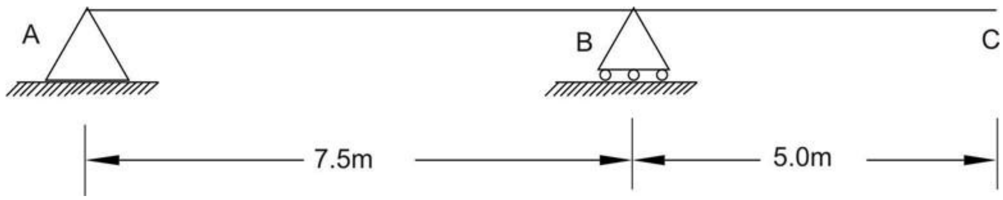


Figure 5: The overhang beam structure

Solution:

As explained earlier in example 1, here we will use tabulated values and influence line equation approach.

Tabulate Values:

As shown in the figure, a unit load is places at distance x from support A and the reaction value R_B is calculated by taking moment with reference to support A. Let us say, if the load is placed at 2.5 m. from support A then the reaction R_B can be calculated as follows.

$$\Sigma M_A = 0 : R_B \times 7.5 - 1 \times 2.5 = 0 \Rightarrow R_B = 0.33$$

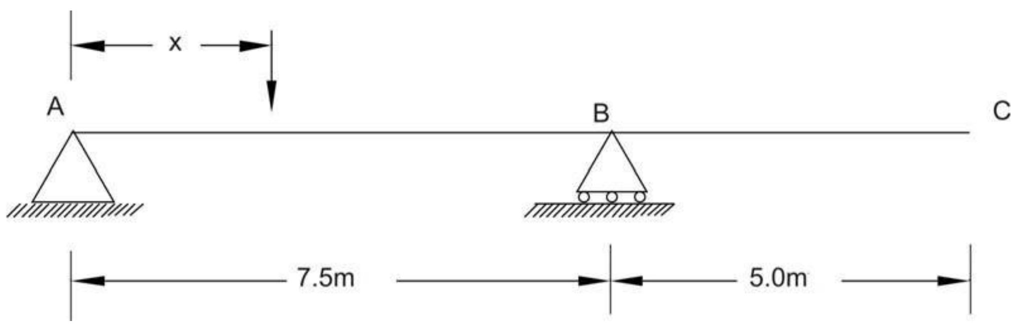


Figure 6: The beam structure with unit load

Similarly one can place a unit load at distances 5.0 m and 7.5 m from support A and compute reaction at B. When the load is placed at 10.0 m from support A, then reaction at B can be computed using following equation.

$$\Sigma M_A = 0 : R_B \times 7.5 - 1 \times 10.0 = 0 \Rightarrow R_B = 1.33$$

Similarly a unit load can be placed at 12.5 and the reaction at B can be computed. The values of reaction at B are tabulated as follows.

x	R_B
0	0.0
2.5	0.33
5.0	0.67
7.5	1.00
10	1.33
12.5	1.67

Graphical representation of influence line for R_B is shown in Figure 7.

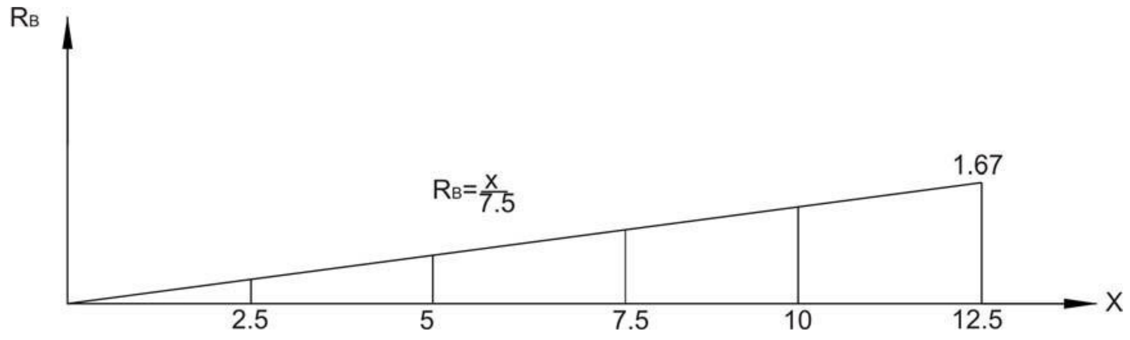


Figure 7: Influence for reaction R_B.

Influence line Equation:

Applying the moment equation at A (Figure 6),

$\Sigma M_A = 0 : R_B \times 7.5 - 1 \times x = 0 \Rightarrow R_B = x/7.5$

The influence line using this equation is shown in Figure 7 on the previous page.

Example 3:

Construct the influence line for shear force at point C of the beam (Figure 8)

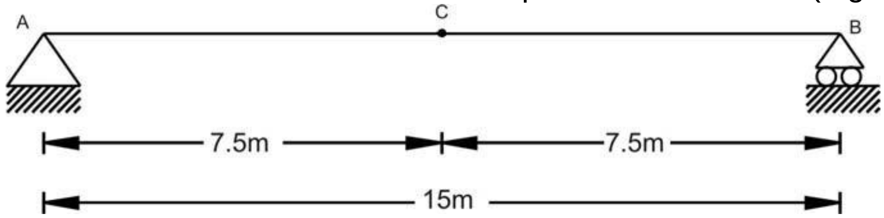


Figure 8: Beam Structure

Solution:

Tabulated Values:

As discussed earlier, place a unit load at different location at distance x from support A and find the reactions at A and finally computer shear force taking section at C. The shear force at C should be carefully computed when unit load is placed before point C (Figure 9) and after point C (Figure 10). The resultant values of shear force at C are tabulated as follows.

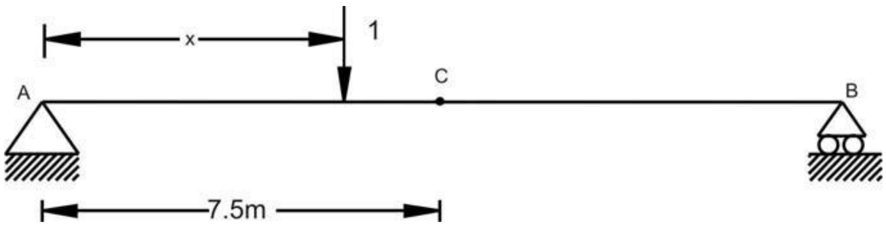


Figure 9: The beam structure – a unit load before section

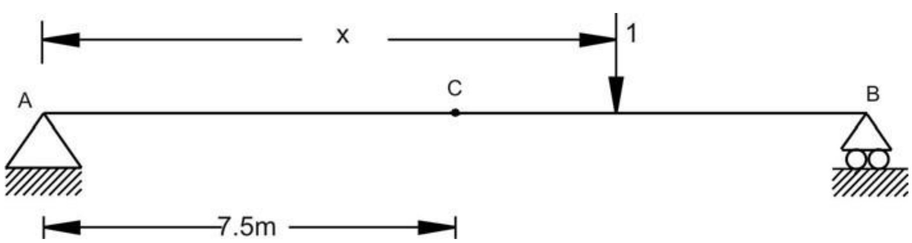


Figure 10: The beam structure - a unit load after section

X	V_c
0	0.0
2.5	-0.16
5.0	-0.33
7.5(-)	-0.5
7.5(+)	0.5
10	0.33
12.5	0.16
15.0	0

Graphical representation of influence line for V_c is shown in Figure 11.

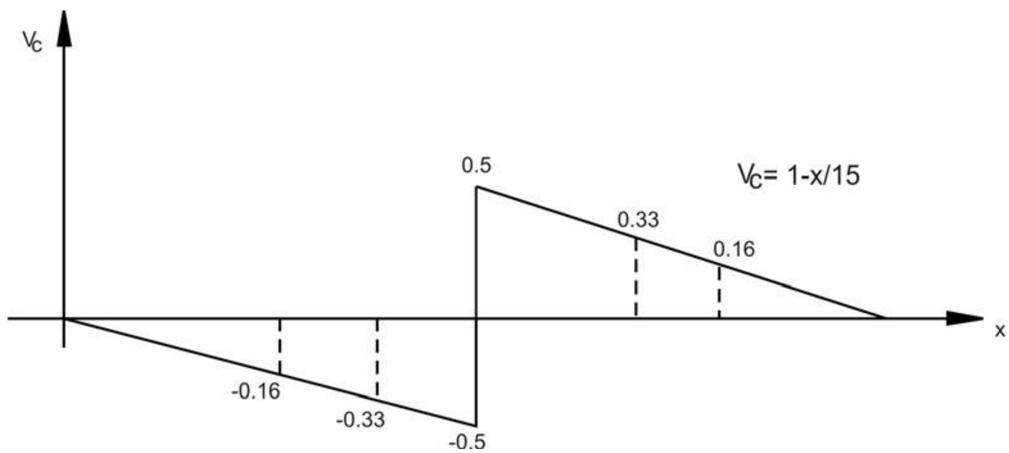


Figure 11: Influence line for shear point C

Influence line equation:

In this case, we need to determine two equations as the unit load position before point C (Figure 12) and after point C (Figure 13) will show different shear force sign due to discontinuity. The equations are plotted in Figure 11.

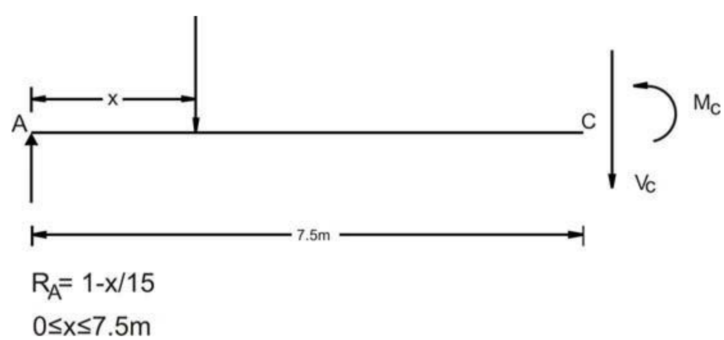


Figure 12: Free body diagram – a unit load before section

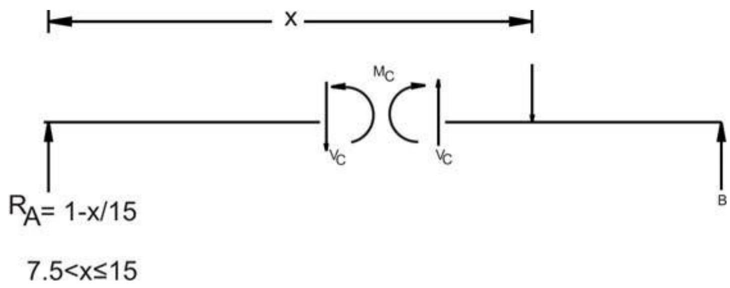


Figure 13: Free body diagram – a unit load after section

Influence Line for Moment:

Like shear force, we can also construct influence line for moment.

Example 4:

Construct the influence line for the moment at point C of the beam shown in Figure 14

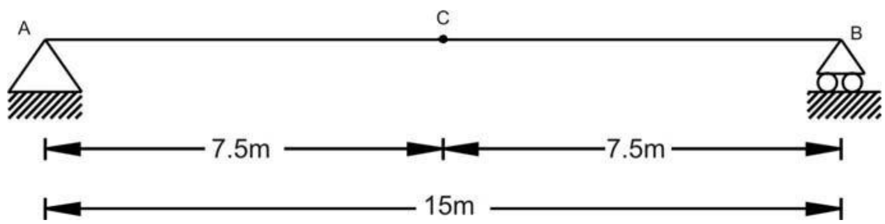


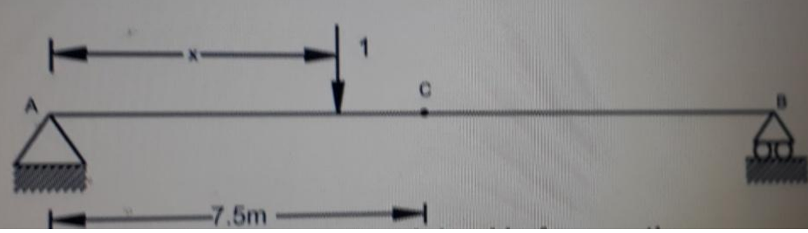
Figure 14: Beam structure

Solution:

Tabulated values:

Place a unit load at different location between two supports and find the support reactions. Once the support reactions are computed, take a section at C and compute the moment. For example, we place the unit load at x=2.5 m from support A (Figure 15), then the support reaction at A will be 0.833 and support reaction B will be 0.167. Taking section at C and computation of moment at C can be given by

$$\Sigma M_c = 0 : -M_c + R_B \times 7.5 - = 0 \Rightarrow -M_c + 0.167 \times 7.5 - = 0 \Rightarrow M_c = 1.25$$



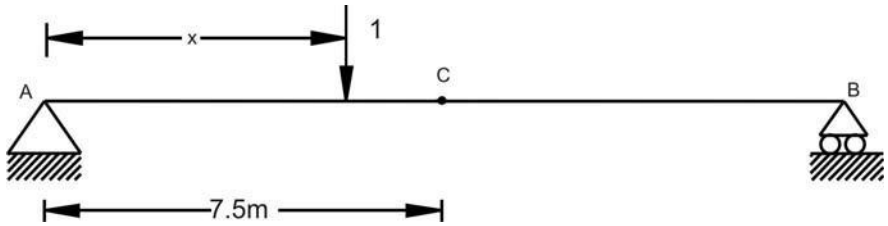


Figure 15: A unit load before section

Similarly, compute the moment M_c for difference unit load position in the span. The values of Mc are tabulated as follows.

X	M_c
0	0.0
2.5	1.25
5.0	2.5
7.5	3.75
10	2.5
12.5	1.25
15.0	0

Graphical representation of influence line for M_c is shown in Figure 16.

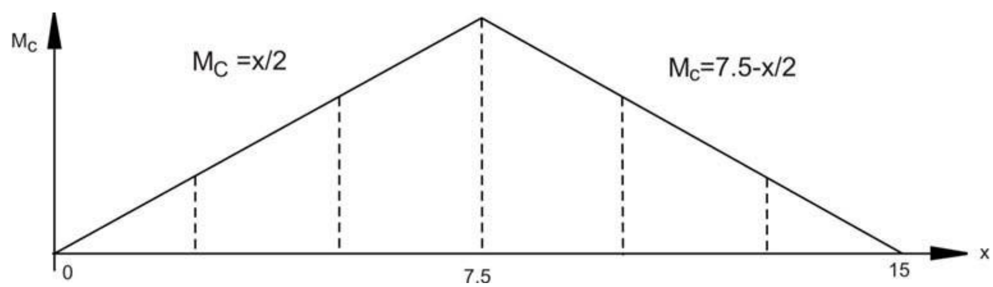


Figure 16: Influence line for moment at section C

Influence Line Equations:

There will be two influence line equations for the section before point C and after point C.

When the unit load is placed before point C then the moment equation for given Figure 37.17 can be given by

$\Sigma M_c = 0 : M_c + 1(7.5 - x) - (1 - x/15)x7.5 = 0 \Rightarrow M_c = x/2$, where $0 \leq x \leq 7.5$

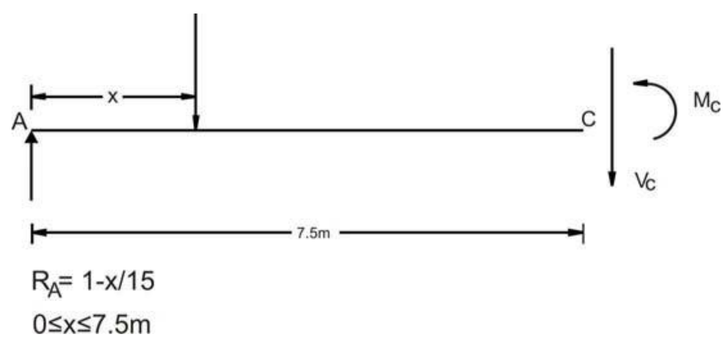


Figure 17: Free body diagram - a unit load before section

When the unit load is placed after point C then the moment equation for given Figure 18 can be given by

$\Sigma M_c = 0 : M_c - (1 - x/15) \times 7.5 = 0 \Rightarrow M_c = 7.5 - x/2$, where $7.5 < x \leq 15.0$

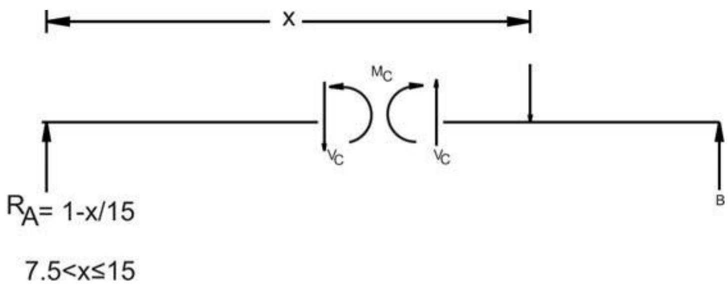


Figure 18: Free body diagram - a unit load after the section

The equations are plotted in Figure 16.

Example 5:
Construct the influence line for the moment at point C of the beam shown in Figure 19.

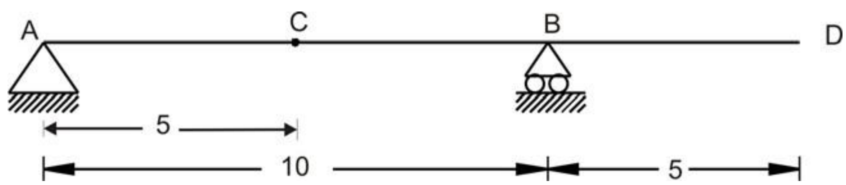


Figure 19: Overhang beam structure

Solution:
Tabulated values:
Place a unit load at different location between two supports and find the support reactions. Once the support reactions are computed, take a section at C and compute the moment. For example as shown in Figure 20, we place a unit load at 2.5 m from support A, then the support reaction at A will be 0.75 and support reaction B will be 0.25.

Figure 20: A unit load before section C

Taking section at C and computation of moment at C is given by

$\Sigma M_c = 0 : \; - M_c + R_B \times 5.0 - \; = 0 \;\Rightarrow\; - M_c + 0.25 \times 5.0 \; = 0 \;\Rightarrow\; M_c = 1.25$

Similarly, compute the moment M_c for different unit load position in the span. The values of Mc are tabulated as follows.

x	M_c
0	0
2.5	1.25
5.0	2.5
7.5	1.25
10	0
12.5	-1.25
15.0	-2.5

Graphical representation of influence line for M_c is shown in Figure 21.

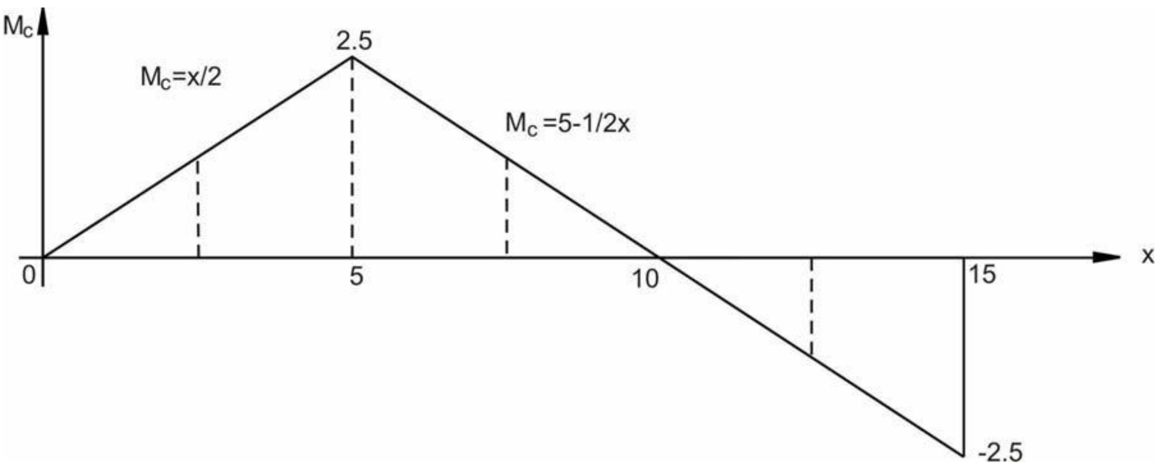


Figure 21: Influence line of moment at section C

Influence Line Equations:

There will be two influence line equations for the section before point C and after point C.

When a unit load is placed before point C then the moment equation for given Figure 22 is given by

$\Sigma M_c = 0 : M_c + 1(5.0 - x) - (1 - x/10)x5.0 = 0 \Rightarrow M_c = x/2$, where $0 \leq x \leq 5.0$

Figure 22: A unit load before section C

When a unit load is placed after point C then the moment equation for given Figure 23 is given by

$\Sigma M_c = 0 : M_c - (1 - x/10) \times 5.0 = 0 \Rightarrow M_c = 5 - x/2$, where $5 < x \leq 15$

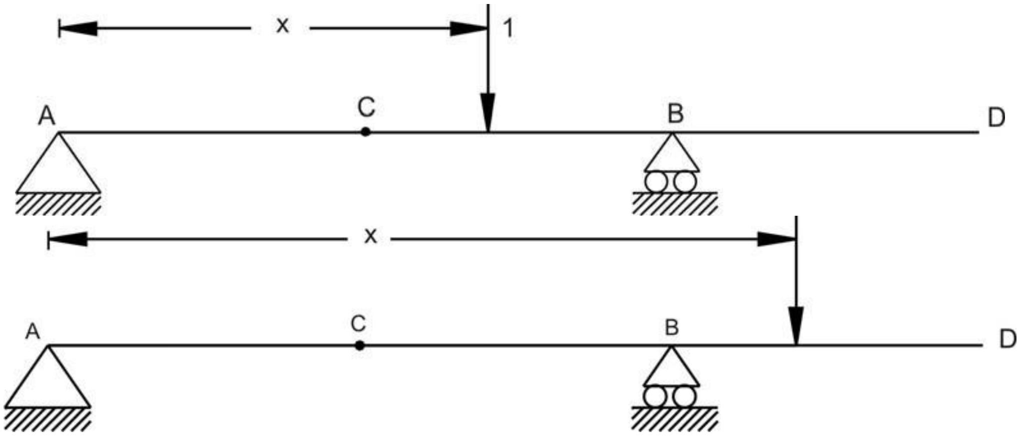


Figure 23: A unit load after section C

The equations are plotted in Figure 21.

Influence line for beam having point load and uniformly distributed load acting at the same time

Generally, beams/girders are main load carrying components in structural systems. Hence it is necessary to construct the influence line for the reaction, shear or moment at any specified point in beam to check for criticality. Let us assume that there are two kinds of load acting on the beam. They are concentrated load and uniformly distributed load (UDL).

Concentrated load

As shown in the Figure 24, let us say, point load P is moving on beam from A to B. Looking at the position, we need to find out what will be the influence line for reaction B for this load. Hence, to generalize our approach, like in earlier examples, let us assume that unit load is moving from A to B and influence line for reaction A can be plotted as shown in Figure 25. Now we want to know, if load P is at the center of span then what will be the value of reaction at A? From Figure 24, we can find that for the load position of P, influence line of unit load gives value of 0.5. Hence, reaction A will be 0.5xP. Similarly, for various load positions and load value, reactions at A are computed.

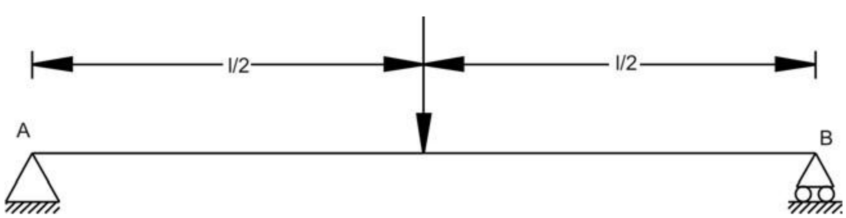


Figure 24: Beam structure

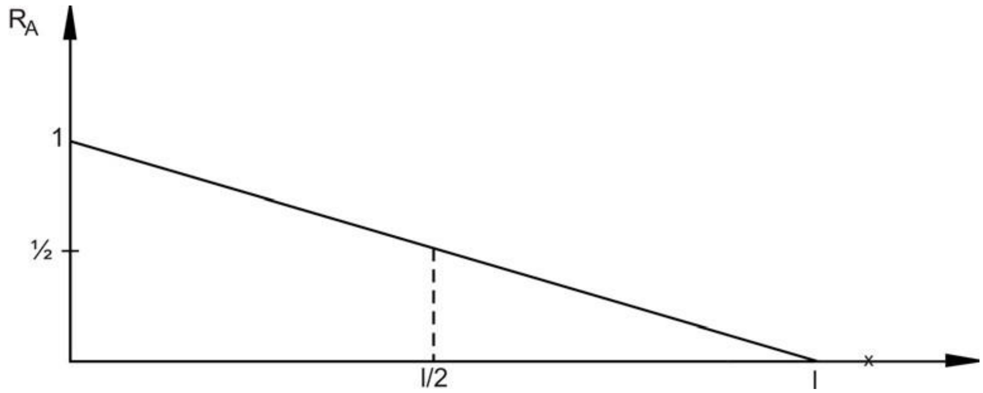


Figure 25: Influence line for support reaction at A

Uniformly Distributed Load

Beam is loaded with uniformly distributed load (UDL) and our objective is to find influence line for reaction A so that we can generalize the approach. For UDL of w on span, considering for segment of dx (Figure 26), the concentrated load dP is given by w.dx acting at x. Let us assume that beam's influence line ordinate for some function (reaction, shear, moment) is y as shown in Figure 27. In that case, the value of function is given by (dP)(y) = (w.dx).y. For computation of the effect of all these concentrated loads, we have to integrate over the entire length of the beam. Hence, we can say that it will be ∫ w.y.dx = w ∫ y.dx. The term ∫ y.dx is equivalent to area under the influence line.

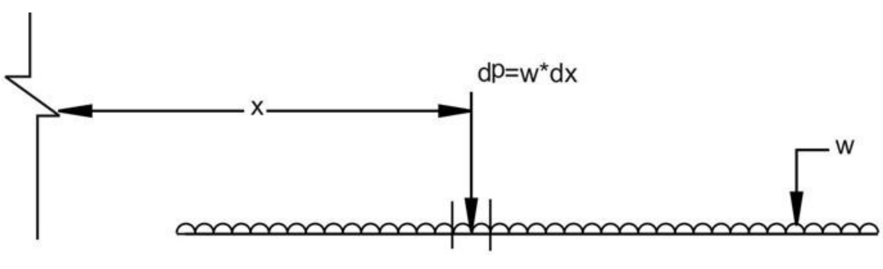


Figure 26: Uniformly distributed load on beam

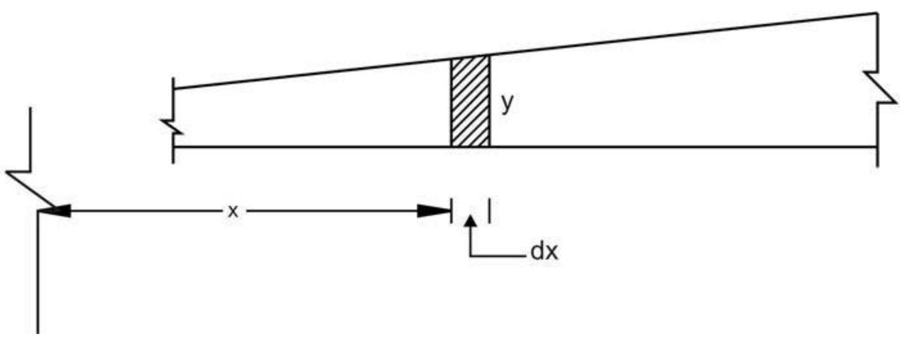


Figure 27: Segment of influence line diagram

For a given example of UDL on beam as shown in Figure 28, the influence line (Figure 29) for reaction at A is given by area covered by the influence line for unit load into UDL value. i.e. [0.5x (1)xl] w = 0.5 w.l.

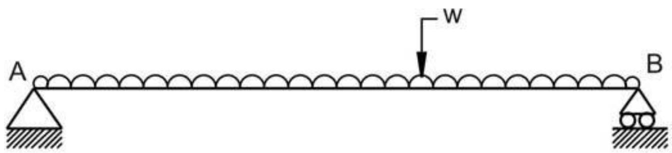


Figure 28: UDL on simply supported beam

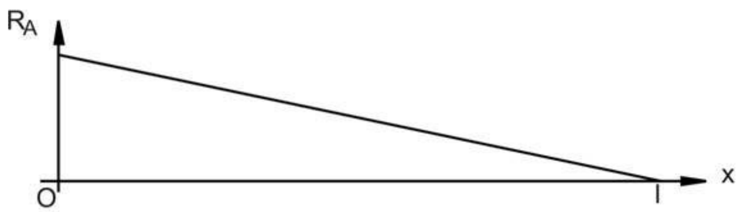


Figure 29: Influence line for support reaction at A.

Numerical Example

Find the maximum positive live shear at point C when the beam (Figure 30) is loaded with a concentrated moving load of 10 kN and UDL of 5 kN/m.

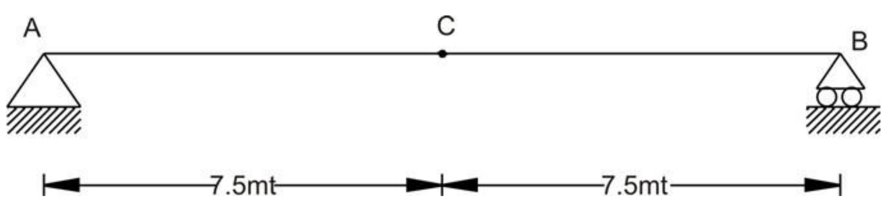


Figure 30: Simply supported beam

Solution:

As discussed earlier for unit load moving on beam from A to B, the influence line for the shear at C is given by following Figure 31.

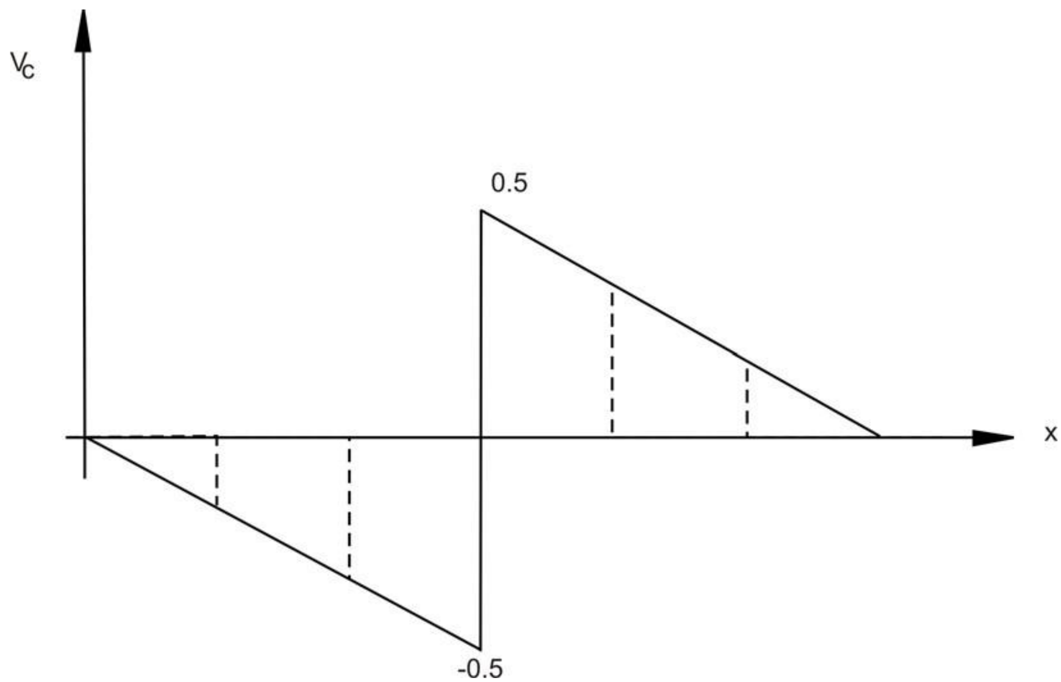


Figure 31: Influence line for shear at section C.

Concentrated load: As shown in Figure 31, the maximum live shear force at C will be when the concentrated load 10 kN is located just before C or just after C. Our aim is to find positive live shear and hence, we will put 10 kN just after C. In that case,

$$V_c = 0.5 \times 10 = 5 \text{ kN.}$$

UDL: As shown in Figure 31, the maximum positive live shear force at C will be when the UDL 5 kN/m is acting between x = 7.5 and x = 15.

$$V_c = [\,0.5 \times (15 - 7.5)\,(0.5)] \times 5 = 9.375$$

Total maximum Shear at C:

$$(V_c)\ max = 5 + 9.375 = 14.375.$$

Finally the loading positions for maximum shear at C will be as shown in Figure 32. For this beam one can easily compute shear at C using statics.

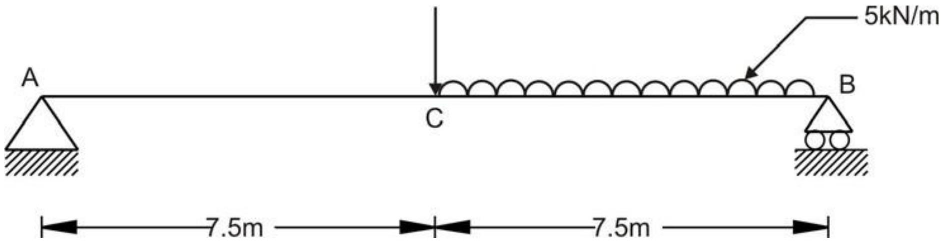


Figure 32: Simply supported beam

Closing Remarks

In this lesson we have studied the need for influence lines and their importance. Further we studied the available various influence line definitions. Finally, we studied the influence line construction using tabulated values and influence line equation. The understanding about the simple approach was studied with the help of many numerical examples.

Suggested Text Books for Further Reading

- Hibbeler, R. C. *Structural Analysis*, Pearson Education (Singapore) Pte. Ltd., Delhi, ISBN 81-7808-750-2

- Junarkar, S. B. and Shah, H. J. *Mechanics of Structures – Vol. II*, Charotar Publishing House, Anand.