Salvage value:-

The salvage value can by obtained from the straight line depreciation by the formula.

$$S = \left(1 - X/L\right)k$$ where X = years of unused life. = years of useful life remaining
L = years of Total life.
k = initial value.

Q1 Example:- A pump is estimated to require replacement after 20 years and is used in a project where the economy study is based on a 50-year period of analysis. What salvage value should be used if the initial cost is \$15,000.

Sol^n After crossing 40 years, the third pump is installed. This pump will have to work only for remaining ten years.
Thus this third pump will have a useful life remaining of ten years more i.e. 10 years of unused life

i.e $X = 10$ years
$L = 20$ years
$k = \$15,000$

$$\therefore S = \left(1 - \frac{10}{20}\right)15,000 \quad \boxed{= \$\,7,500}$$

Dynamic programming :-

For water allocation in the previous problem where θ is the water quantity alloted to each three users $j = 1, 2, 3$, we had to determine the allocation x_j to each user j. We had also to maximise the benefits for each user.

Now instead of analysing the benefit to a particular user, we shall study the total benefits to the project. Let us assume that the gross benefits are defined by the function $a_j[1 - e^{-b_j x_j}]$ and cost defined by the function $c_j x_j^{d_j}$ where c_j and D_j are known. The problem would then become

$$\text{Maximize} \sum_{j=1}^{3} \{a_j[1 - \exp(-b_j x_j)] - c_j x_j^{d_j}\} \quad \text{---} (1)$$

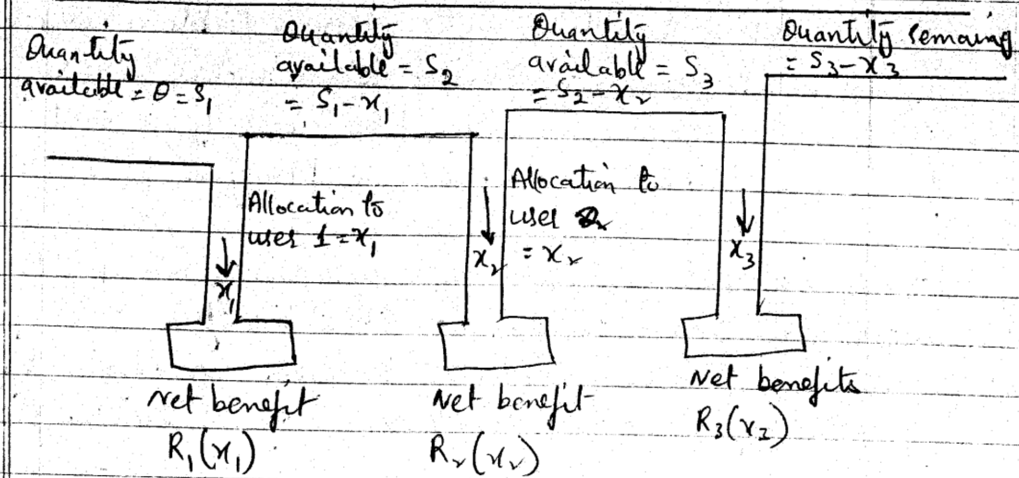
subject to the constraint

$$\sum_{j=1}^{3} x_j \leq \theta \quad \text{and} \quad x_j \geq 0 \text{ for each user}$$

Let us assume three users, i.e. $j = 1, 2, 3$.

also let S_1, S_2, S_3, S_4 be the water quantity available at various stages after distribution to a consumer.
let x_1, x_2, x_3 represent the allocation to each user.
let $R_1(x_1)$, $R_2(x_2)$, $R_3(x_3)$ denote the net benefits to users 1, 2 and 3. An allotment diagram is given:

class 15



Allocation to each user is considered as a decision stage. When a portion of water is alloted (say x_j) out of the total water quantity (θ), this produces a net benefit given by $R_j(x_j) = a_j\left[1 - e^{-b_j x_j}\right] - c_j x_j^{d_j}$ --- (2)

i considering eqns (1) & (2) we can write

$$f_1(\theta) = \text{maximum}\left[R_1(x_1) + R_2(x_2) + R_3(x_3)\right] \quad \text{--- (3)}$$

with the condition that $x_1 + x_2 + x_3 \leq \theta$.

$x_1, x_2, x_3 \geq 0$. (No negative supply)

⇒ ($f_1(\theta)$ is the total net benefit that can be obtained by supplying water to three users)

It can further be written as

$$f_1(\theta) = \text{Max}\{R_1(x_1) + \text{Max}\{R_2(x_2) + \text{Max}\{R_3(x_3)\}\} \quad \text{--- (4)}$$

starting from user 3, the quantity available = S_3
If $f_3(S_3)$ equals the max. benefit to user 3 when a quantity of water S_3 is available, Then

$$f_3(S_3) = \max[\underbrace{R_3(x_3)}_{\text{Net benefit}}] \quad \text{------} (5)$$

also $S_3 = S_2 - x_2$,

$\therefore f_3(S_2 - x_2) = \max[R_3(x_3)]$. put this value in
eqn (4) we get :

$$f_1(Q) = \max[R_1(x_1)] + \max[R_2(x_2)] + f_3(S_2 - x_2)\}$$

also $\quad f_2(S_2) = \max[R_2(x_2) + f_3(S_2 - x_2)] \quad \text{---} (6)$

$$\therefore f_1(Q) = \max[R_1(x_1)] + f_2(S_2)$$

But $S_2 = Q - x_1$

$$\therefore f_1(Q) = \max[R_1(x_1)] + f_2(Q - x_1)] \quad \text{---} (7)$$

Thus the eqn is reduced to a simple form. eqns (5), (6), (7) are called as (Recursive sets of eqns). From (5) we can find $F_3(S_3)$, put in (6) and find $f_2(S_2)$ from these. Put value of $f_2(S_2)$ in (7), we get $f_1(Q)$.

class 16

MF 2nd final

Benefit - cost ratio (Problem:-) Refer note

The capital cost of an irrigation project at the end of its construction period is Rs 50 (55) lacs. The value of gross benefits from the project = Rs 5 (5.5) lacs per year where as the increase in the cost of agricultural operations is Rs 1.5 (1.4) lacs per year. The annual cost of operation, maintanance and repair = Rs 50,000 (55,000)
Assuming that the benefits from the project will begin 10 years after completion and will occur uniformly over the remaining 65 years of the estimated useful life of 75 years. ~~Determine benefit ratio of the project~~
Determine the benefit-cost ratio. Assume rate of interest = 3 %.

Total annual cost = $(Ci + O + D)$

Solution:- capital cost $(C) = 50,000,00$

rate of interest $(i) = 3\%$

operation, maintanance and repair cost $(O) = 50,000$

Now, Total annual cost $= (Ci + O + D)$

where D = depreciation $= \dfrac{Ci}{(1+i)^n - 1}$

n = estimated life span = 75 years

$\therefore$ Total annual cost $= \left(50,000,00 \times \dfrac{3}{100} + 50,000 + \dfrac{50,000,00 \times \frac{3}{100}}{\left(1+\frac{3}{100}\right)^{75} - 1}\right.$

$= 50{,}000{,}00 \times 0.03 + 50000 + \dfrac{50{,}000{,}00 \times 0.03}{(1.03)^{75} - 1}$

$= 15{,}0000 + 50000 + \dfrac{15{,}0000}{9.1789 - 1}$

$=$ Rs

$\therefore$ Total annual cost $=$ Rs $\boxed{218340=00}$

calculation of Net Annual benefits :-

Benefits start to come after 10 years of completion

$\therefore$ [illegible] annual income from years 11 to 75

Net Benefit $(R) = (5.00 - 1.50) =$ Rs 3.50 lacs $=$ Rs 350000

(Net benefit)

Now, present value (worth) of Rs 3.50 lacs at the begining of 11th year when benefits start coming $= R(P/A, i, n)$ — applying discounting factors

$= R\left[\dfrac{(1+i)^n - 1}{(1+i)^n i}\right]$

$PW_1 = R \times (P/A, i, n)$

$= R\left[\dfrac{(1+0.03)^{65} - 1}{(1.03)^{65} \times 0.03}\right] = 350000\left[\dfrac{1 - (1+0.03)^{-65}}{0.03}\right]$

$= R\left[\dfrac{1 - \frac{1}{(1+0.03)^{65}}}{0.03}\right] = 350000\left[\dfrac{1 - 0.1464}{0.03}\right]$

$\boxed{PW_1 = \text{Rs } 9958550}$

$\boxed{(P/A, i, n) = \dfrac{(1+i)^n - 1}{i(1+i)^n}}$

Present worth of above amount net benefits at the end of construction period

i.e. $= PW_2 = PW_1 \times (P/F, i, n) = PW_1 \times \dfrac{1}{(1+i)^n} \Rightarrow \left[(P/F, i, n) = \dfrac{1}{(1+i)^n}\right]$

$PW_2 = \dfrac{9958550}{(1+0.03)^n} = \dfrac{9958550}{(1.03)^{10}} =$ Rs $\underline{7410157}$

Spreading the benefits of 65 years over a period of 75 years
i.e annual equivalent net benefits over 75 years

$$= \frac{PW_2}{(P/A, i, n)}$$

$$(P/A, i, n) = \frac{(1+i)^n - 1}{(1+i)^n i}$$

$$PW = \frac{PW_2}{\frac{(1+i)^n - 1}{(1+i)^n i}} = PW_2 \frac{(1+i)^n \cdot i}{(1+i)^n - 1}, \quad i = 3\% = 0.03, \; n = 75 \text{ yrs}$$

$$= \frac{7410157 (1.03)^{75} \times 0.03}{(1.03)^{75} - 1}$$

$$= 249472$$

$\therefore$ Benefit cost ratio $= \frac{249472}{218340} = 1.14$