

DEPARTMENT OF CIVIL ENGINEERING

# LAB MANUAL

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STRUCTURAL ANALYSIS

NATIONAL INSTITUTE OF TECHNOLOGY |  
SRINAGAR J&K

# **NATIONAL INSTITUTE OF TECHNOLOGY, SRINAGAR**

## **Vision of the Institute**

To establish a unique identity of a pioneer technical Institute by developing a high-quality technical manpower and technological resources that aim at economic and social development of the nation as a whole and the region in particular keeping in view the global challenges.

## **Mission of the Institute**

- M1.** To create a strong and transformative technical educational environment in which fresh ideas, moral principles, research and excellence nurture with international standards.
  
- M2.** To prepare technically educated and broadly talented engineers, future innovators and entrepreneurs, graduates with understanding of the needs and problems of the industry, the society, the state and the nation.
  
- M3.** To inculcate the highest degree of confidence, professionalism, academic excellence and engineering ethics in budding engineers.

## **VISION OF THE DEPARTMENT**

To nurture Civil engineers with passion for professional excellence, ready to take global challenges and to serve the society with high human values.

## **MISSION STATEMENT OF THE DEPARTMENT**

- (1)** To provide facilities and infrastructure for academic excellence in the field of Civil engineering.
- (2)** To inculcate in the student the passion for understanding professionalism, ethics, safety, sustainability and then actively contribute in the society.
- (3)** To nurture creativity and encourage innovative solutions to real life challenging problems in Civil engineering students.
- (4)** To prepare student for lifelong learning in global perspective.

## **PROGRAM EDUCATIONAL OBJECTIVES**

PEO1: To prepare students to get employment, profession and/or to pursue post-graduation and research in Civil engineering discipline in particular and allied engineering fields in general.

PEO2: To prepare students to identify and analyse Civil engineering problems in an iterative approach that involves defining, quantifying, testing and review of the identified problem.

PEO3: To prepare students to plan, organize, schedule, execute and communicate effectively as an individual, a team member or a leader in multidisciplinary environment.

PEO4: To provide the students, an academic environment that makes them aware of excellence in field of Civil Engineering and enables them to understand significance of lifelong learning in global perspective.

## INDEX

1. To determine Flexural Rigidity ( $EI$ ) of a given beam
2. To verify Maxwell's Reciprocal theorem.
3. To find horizontal thrust in a three-hinged arch and to draw influence line diagrams for Horizontal Thrust and Bending moment.
4. To find horizontal thrust in a two hinged arch and to draw influence line diagrams for horizontal Thrust and bending moment.
5. To find Critical load in struts with different end conditions.
6. To find deflections in Beam having unsymmetrical bending.
7. To find the determination of elastic deflection of curved beams.
8. To analysis the redundant joint.

## Experiment No. 1

### Flexural rigidity

**Aim:** - To find the value of flexural rigidity (EI) for a given beam and compare it with theoretical value

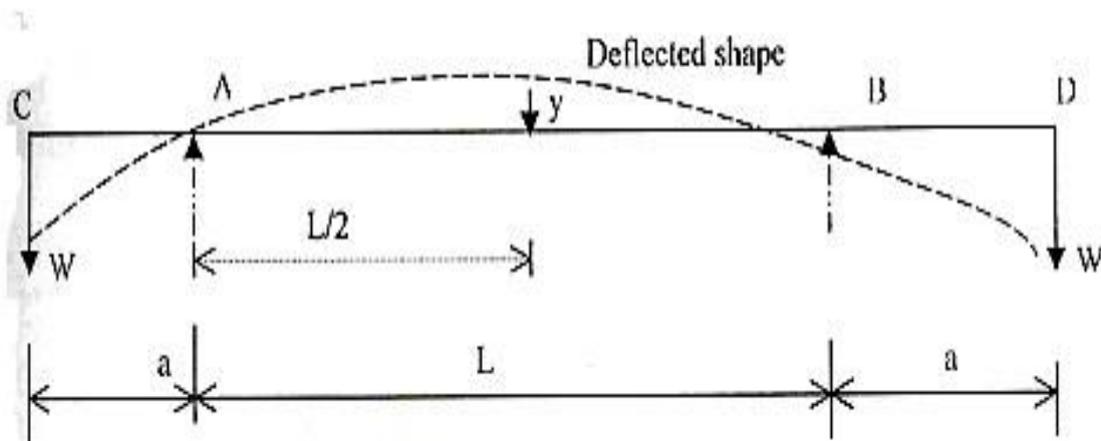
**Apparatus:** - Elastic Properties of deflected beam, weight's, hanger, dial gauge, scale, and Verniar caliper.

**Formula:** - (1) Central upward deflection,  $y = W.a.L^2 / 8y$  ..... (1)

(2)  $EI = W.a. L^2 / 8y$  ..... (2)

(3) Also it is known that EI for beam =  $E \times bd^3 / 12$  ... ..... (3)

Diagram:-



**Theory:** - For the beam with two equal overhangs and subjected to two concentrated loads W each at free ends, maximum deflection y at the centre is given by central upward deflection. Central upward deflection,  $y = W.a.L^2 / 8EI$

Where,

a = length of overhang on each side

W = load applied at the free ends

L = main span

E = modulus of elasticity of the material of the beam

I = moment of inertia of cross section of the beam

$$EI = W.a.L^2 / 8y$$

It is known that, EI for beam =  $E \times bd^3 / 12$

Where, b = width of beam

d = depth of beam

**Procedure: -**

- i) Find b and d of the beam and calculate the theoretical value of EI by Eq. (3).
- ii) Measure the main span and overhang span of the beam with a scale.
- iii) By applying equal loads at the free end of the overhang beam, find the central deflection y.
- iv) Repeat the above steps for different loads.

**Observation: -** 1) Length of main span, L (cm) =

2) Length of overhang on each side, a (cm) =

3) Width of beam, b (cm) =

4) Depth of beam, d (cm) =

5) Modulus of elasticity, E (kg/cm<sup>2</sup>) =  $2 \times 10^6$

**Observation Table:-**

Sr. No	. Equal loads at the two ends (kg)	Dial gauge reading at the midspan of beam(cm)	EI from Eq. ( 3 )	EI from Eq ( 2 )

**Calculation: -** Average values of EI from observation = .....cm<sup>4</sup>

Average values of EI from calculation = .....cm<sup>4</sup>

**Result: -** Flexural rigidity (EI) is found same theoretically and experimentally.

- Precaution:-**
- 1. Measure the center deflection y very accurately.
  - 2. Ensure that the beam is devoid of initial curvature.
  - 3. Loading should be within the elastic limit of the materials.

**Question:-**

1. What is the unit of flexural rigidity?
2. Which types of beam are used in deflected beam apparatus?
3. Define the size of beam which is used in deflected beam apparatus.
4. What is the difference b/w flexural rigidity and flexural stiffness?
5. What is flexural rigidity?

## Experiment No.2

### Maxwell's reciprocal theorem

**Aim:** - To verify Clerk Maxwell's reciprocal theorem

**Apparatus:** - Clerk Maxwell's Reciprocal Theorem apparatus, Weight's, Hanger, Dial Gauge, Scale Vernier caliper.

**Diagram:-**



**Theory:** -

Maxwell theorem in its simplest form states that deflection of any point A of any elastic structure due to load P at any point B is same as the deflection of beam due to same load applied at A. It is, therefore easily derived that the deflection curve for a point in a structure is the same as the deflected curve of the structure when unit load is applied at the point for which the influence curve was obtained.

**Procedure:** -

- i) Apply a load either at the centre of the simply supported span or at the free end of the beam, the deflected form can be obtained.
- ii) Measure the height of the beam at certain distance by means of a dial gauge before and after loading and determine the deflection before and after at each point separately.
- iii) Now move a load along the beam at certain distance and for each position of the load, the deflection of the point was noted where the load was applied in step 1. This deflection should be measured at each such point before and after the loading, separately.
- iv) Plot the graph between deflection as ordinate and position of point on abscissa the plot for graph drawn in step 2 and 3. These are the influence line ordinates for deflection of the beam.

**Observation Table:-**

Distance from the pinned end	Load at central point/cantilever end		Deflection of various points (mm) 2-3	Load moving along beam		Deflection of various points (mm) 5-6
	Beam unloaded Dial gauge reading (mm) <sup>2</sup>	Beam loaded Dial gauge reading (mm) <sup>3</sup>	Beam unloaded Dial gauge reading (mm) <sup>5</sup>	Beam unloaded Dial gauge reading (mm) <sup>5</sup>	Beam loaded Dial gauge reading (mm) <sup>6</sup>	

**Result:** - The Maxwell reciprocal theorem is verified experimentally and analytically.

**Precaution:** - (i) Apply the loads without any jerk.

(ii) Perform the experiment at a location, which is away from any

(iii) Avoid external disturbance.

(iv) Ensure that the supports are rigid.

**Question:-**

1. What is the Maxwell's reciprocal theorem or define the Maxwell's reciprocal theorem?
2. What are the purpose of providing dial gauge and magnetic base in the apparatus?
3. Maxwell reciprocal theorem in structural analysis can be applied in-  
A. all elastic structures    B. plastic structure    C. symmetrical structures only  
D. prismatic element structure only
4. What is the difference B/W Maxwell's reciprocal theorem and betties

## Experiment: 3

### Three hinged arch

#### Aim:

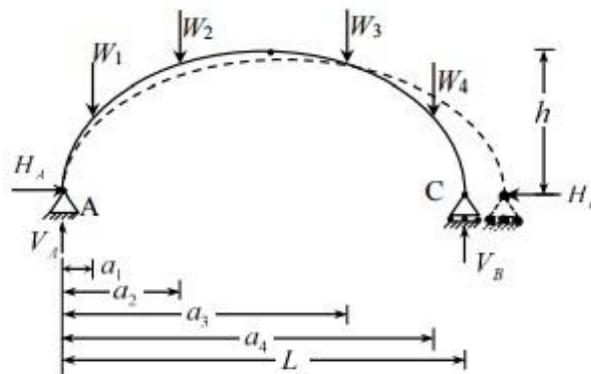
To study the behaviour of a three hinged arch experimentally for the horizontal and vertical displacement of the roller end for a given system of loading and to compare the same with the results obtained by analytical calculations.

#### Apparatus:

Three hinged arch apparatus, weights, scale, dial gauge, etc.

#### Theory:

A three hinged arch is a determinate structure with the axial thrust assisting in maintaining the stability. The horizontal thrust  $H$  in the arch for a number of loads can be obtained as follows



The reactions  $V_A$  and  $V_B$  are calculated using the following equations:

$$V_A = \frac{[W_1(L - a_1) + W_2(L - a_2) + W_3(L - a_3) + W_4(L - a_4)]}{L}$$

$$V_B = \frac{[W_1a_1 + W_2a_2 + W_3a_3 + W_4a_4]}{L}$$

$$H_A + H_B = 0$$

$$V_A + V_B = W_1 + W_2 + W_3 + W_4$$

Take Moment about the hinge C

$$H = \frac{1}{h} [V_s f - W_1 (a_1 - \frac{h}{2})]$$

**Procedure:**

- i. Use lubricating oil at the roller end of the arch so as to have a free movement of the roller end.
- ii. Balance the self-weight of the arch by placing load on hanger for horizontal thrust until the equilibrium conditions is obtained. Under this condition, the roller end of the arch has a tendency to move inside on tapping the table. Note down the load in kg.
- iii. Place a few loads on the arch in any chosen positions. Balance these by placing additional weights on the hanger for horizontal thrust. The additional weights on the thrust hanger give the experimental value of the horizontal thrust.

**Observation:**

Span of the arch,  $L =$

Central rise,  $h =$

Initial load on the thrust hanger for balancing,  $=$

S. No.	Load Applied on Hanger kg	Distance from Left hand Support cm	Additional load on thrust hanger	Calculated value of H
1	$W_1$	$a_1$		
	$W_2$	$a_2$		
	$W_3$	$a_3$		
	$W_4$			
2	$W_1$	$a_1$		
	$W_2$	$a_2$		
	$W_3$	$a_3$		
	$W_4$			

**Calculation:**

**Precautions:**

- i. Put the weights in thrust hanger very gently without a jerk.
- ii. Measure the distance of loaded points from left hand support accurately. Perform the experiment away from vibration and other disturbances.

**Results:**

- i. Find the horizontal thrust for a given set of load experimentally and theoretically.

Experimental value of horizontal thrust,  $H_z$ ,  $=$

Theoretical value of horizontal thrust,  $H_t$ ,  $=$

**Comments:**

**Question:-**

- Define two hinge arches?
- What is the main difference in three hinged arch and two hinge arch?
- Three hinge arch structure are- (a) Determinate structure (b) Indeterminate Structure.
- The bending moment of three hinge arch is greater than the bending moment of beam. (a) True (b) False.
- Write the expression for bending moment at any section on the arch. Given-  $V_A$ = Vertical load at point A ,  $W_1$ = load from end A at distance a, H= horizontal force at point A ,  $x$ = e/s distance from A
- ON the basis of support conditions arches are classified / classified the arches on the basis support conditions.

## Experiment No. 4

### Two hinged arch

**Aim:** - To study two hinged arch for the horizontal displacement of the roller end for a given system of loading and to compare the same with those obtained analytically.

**Apparatus:** - Two Hinged Arch Apparatus, Weight's, Hanger, Dial Gauge, Scale, Verniar Caliper.

**Formula:** -  $H = 5WL (a - 2a^3 + a^4)/8r$

Where,

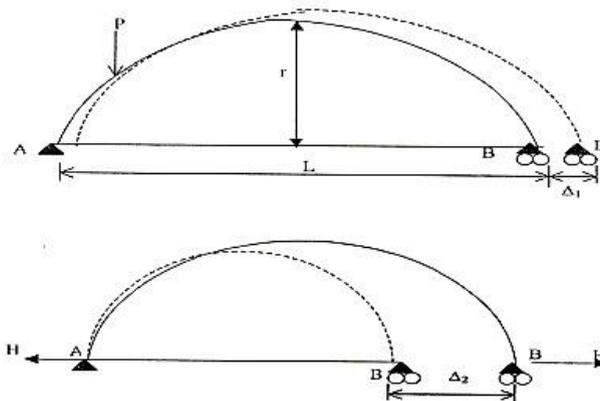
W= Weight applied at end support.

L= Span of two hinged arch.

r= rise of two hinged arch.

a = dial gauge reading.

**Diagram:-**



**Theory:** - The two hinged arch is a statically indeterminate structure of the first degree. The horizontal thrust is the redundant reaction and is obtained by the use of strain energy methods. Two hinged arch is made determinate by treating it as a simply supported curved beam and horizontal thrust as a redundant reaction. The arch spreads

out under external load. Horizontal thrust is the redundant reaction is obtained by the use of strain energy method.

**Procedure: -**

i) Fix the dial gauge to measure the movement of the roller end of the model and keep the lever out of contact.

ii) Place a load of 0.5kg on the central hanger of the arch to remove any slackness and taking this as the initial position, set the reading on the dial gauge to zero.

iii) Now add 1 kg weights to the hanger and tabulated the horizontal movement of the roller end with increase in the load in steps of 1 kg. Take the reading up to 5 kg load. Dial gauge reading should be noted at the time of unloading also.

iv) Plot a graph between the load and displacement (Theoretical and Experimental) compare. Theoretical values should be computed by using horizontal displacement formula.

v) Now move the lever in contact with 200gm hanger on ratio 4/1 position with a 1kg load on the first hanger. Set the initial reading of the dial gauge to zero.

vi) Place additional 5 kg load on the first hanger without shock and observe the dial gauge reading.

vii) Restore the dial gauge reading to zero by adding loads to the lever hanger, say the load is  $w$  kg.

viii) The experimental values of the influence line ordinate at the first hanger position shall be  $4w$ .

ix) Repeat the steps 5 to 8 for all other hanger loading positions and tabulate. Plot the influence line ordinates.

x) Compare the experimental values with those obtained theoretically by using equation (5).

**Observation Table: -** Horizontal displacement

Sr. No.	Central load (kg)	0.0	1.0	2.0	3.0	4.0	5.0	6.0
	Observed horizontal Displacement ( mm )							
	Calculated horizontal Displacement Eq. (4)							

**Sample Calculation:** - Central load (kg) =.....

Observed horizontal Displacement (mm) =

Calculated horizontal Displacement =  $H = 5WL (a - 2a^3 + a^4)/8r = \dots\dots\dots$

**Result:-**The observed and horizontal displacement is nearly same.

**Precaution:** - 1.Apply the loads without jerk.

2. Perform the experiment away from vibration and other disturbances.**Question:-**

1. Define two hinged arch.
2. Two hinged arch structure are-
  - a. Determinate structure
  - b. indeterminate structure
3. What is the application of two hinged arch?
4. Two hinged arch construction are complicated – TRUE/FLASE
5. The load carrying capacity of two hinge arch is higher than beams- TRUE/FLASE
6. The bending moment of arch is higher than the B.M. of beam- TRUE/FLASE

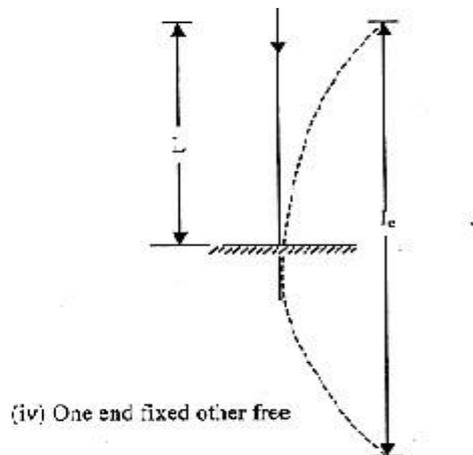
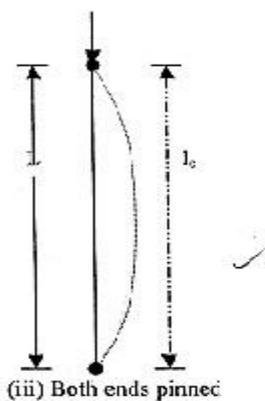
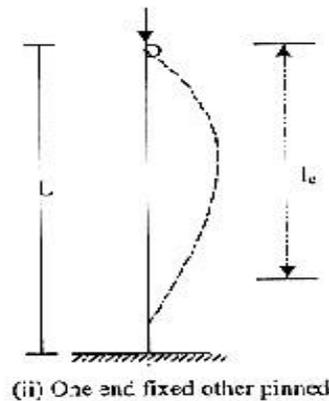
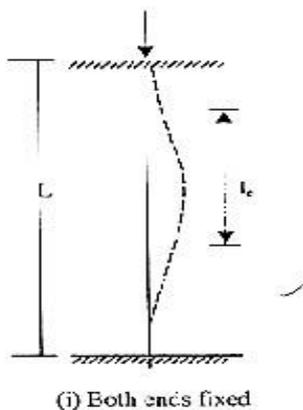
## Experiment No 5

### Column and buckling

**Object:** - To study behavior of different types of columns and find Euler's buckling load for each case.

**Apparatus:** - Column Buckling Apparatus, Weights, Hanger, Dial Gauge, Scale, Vernier caliper.

**Diagram:-**



**Theory:** -If compressive load is applied on a column, the member may fail either by crushing or by buckling depending on its material, cross section and length. If member is considerably long in comparison to its lateral dimensions it will fail by buckling. If a member shows signs of buckling the member leads to failure with small increase in load. The load at which the member just buckles is called as crushing load. The buckling load, as given by Euler, can be found by using following expression.

$$P = \pi^2 EI/le^2$$

Where,

E = Modulus of Elasticity=  $2 \times 10^5 \text{ N/mm}^2$  for steel

I = Least moment of inertia of column section

Le = Effective length of column

Depending on support conditions, four cases may arise. The effective length for each of which are given as:

1. Both ends are fixed  $le = L/2$
2. One end is fixed and other is pinned  $le = L/\sqrt{2}$
3. Both ends are pinned  $le = L$
4. One end is fixed and other is free  $le = 2L$

**Procedure: -**

- i) Pin a graph paper on the wooden board behind the column.
- ii) Apply the load at the top of columns increasing gradually. At certain stage of loading the columns shows abnormal deflections and gives the buckling load.
- iii) Note the buckling load for each of the four columns.
- iv) Trace the deflected shapes of the columns over the paper. Mark the points of change of curvature of the curves and measure the effective or equivalent length for each case separately.
- v) Calculate the theoretical effective lengths and thus buckling loads by the expressions given above and compare them with the observed values.

**Observation: -**

- 1) Width of strip (mm) b =
- 2) Thickness of strip (mm) t =
- 3) Length of strip (mm) L =
- 4) Least moment of inertia

$$I = bt^3/12$$

**Observation Table:-**

Sr. No	End condition	Euler's Buckling load ( $P = \pi^2 EI$ )		Effective Length (mm)	
		Theoretical	Observed	Theoretical	Observed
1.	Both ends fixed				

2.	One end fixed and other pinned				
3.	Both ends pinned				
4.	One end fixed and other free.				

**Sample Calculation:** - Both ends fixed

Euler's buckling load. =  $le^2$

Effective Length (mm) =.

**Result:-**The theoretical and experimental Euler's buckling load for each case is found nearly same.

**Question:-**

1. Define buckling?
2. What is the difference b/w buckling and twisting?
3. What is the difference b/w column and strut?
4. Define buckling facture?

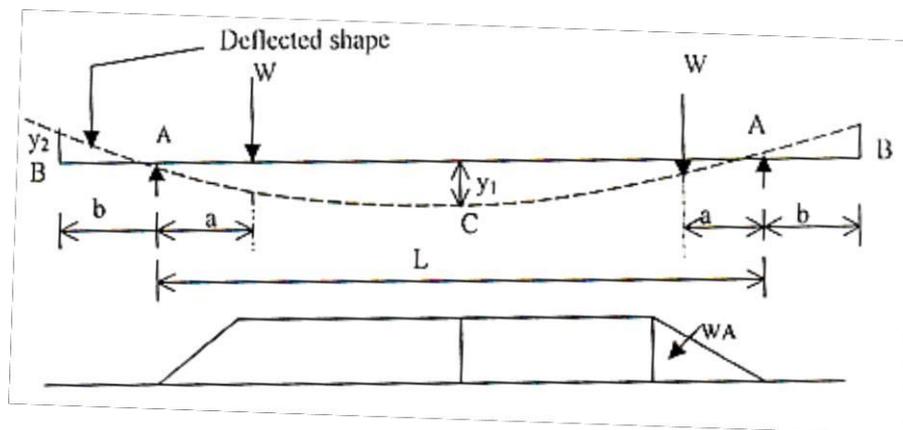
## Experiment No-6

### Deflections of beam

**Aim:** - To verify the moment area theorem regarding the slopes and deflections of the beam.

**Apparatus:** - Moment of area theorem apparatus.

**Diagram:-**



**Fig.-**

**Theory :** - According to moment area theorem

1. The change of slope of the tangents of the elastic curve between any two points of the deflected beam is equal to the area of  $M/EI$  diagram between these two points.
2. The deflection of any point relative to tangent at any other point is equal to the moment of the area of the  $M/EI$  diagram between the two points at which the deflection is required. Slope at B =  $Y_2 / b$  since the tangent at C is horizontal due to symmetry,

$$\text{Slope at B} = \text{shaded area} / EI = 1 / EI [Wa^2 / 2 + WA (L/2 - a)]$$

Displacement at B with respect to tangent at C

$$= (y_1 + y_2) = \text{Moment of shaded area about B} / EI$$

$$= 1 / EI [Wa^2 / 2 (b + 2/3a) + Wa(L/2 - a) (b + a/2 + L/2)]$$

**Procedure:** -

1. Measure a, b and L of the beam
2. Place the hangers at equal distance from the supports A and load them with equal loads.
3. Measure the deflection by dial gauges at the end B ( $y_2$ ) and at the center C ( $y_1$ )
4. Repeat the above steps for different loads.

**Observation Table:-**

Length of main span, L (cm) =

Length of overhang on each side, a (cm) =

Modulus of elasticity, E (kg/cm<sup>2</sup>) =  $2 \times 10^6$

Sl. No.	Load at each Hanger (kg)	Central Deflection Y <sub>1</sub> (cm)	Deflection at Free end y <sub>2</sub> (cm)	Slope at B Y <sub>2</sub> / b	Deflection at C=Deflection atB (y <sub>1</sub> )

**Calculation:-**

1. Calculate the slope at B as  $y_2 / b$  (measured value).
2. Compute slope and deflection at B theoretically from B.M.D. and compare with experimental values.
3. Deflection at C =  $y_1$ (measured value).
4. Deflection at C = Average calculated value

**Result:** -The slope and deflection obtained is close to the slope and deflection obtained by using moment area method.

**Precaution:-**

1. Apply the concentration loads without jerks.
2. Measures the deflection only when the beam attains ion.
3. Measures the deflection very carefully and accurately.
4. Check the accuracy and least count of dial gauges used for measuring deflections.

## Experiment:-7

### Deflection of curved members

#### **Aim:**

To determine the elastic displacement of the curved members experimentally and verify the same with the analytical results.

#### **Apparatus:**

Curved beam apparatus with four different types of configurations, weights, scale, dial gauges and Vernier Caliper.

#### **Theory:**

The elastic displacements of a curved member can be determined using Castigliano's first theorem which states that "The partial derivative of the strain energy with respect to any force gives the displacement of the point of its application in the direction of the force."

The total strain energy of any structure is determined in terms of the entire load with their actual values and a fictitious load  $P$  applied at the point at which the deflection is required and it is acting in the same direction in which the deflection is required. In case no external load is acting at the joint in the direction desired, a fictitious load is applied in that direction and forces in all the members are worked out. After partial differentiation with respect to  $P$ , zero is substituted for the fictitious load  $P$  (or if  $P$  is not fictitious its actual value is substituted). Thus the result is the required deflection.

#### *a. Quadrant of a circle*

The curved beam is fixed at the point A and is free at point B. The concentrated load,  $P$  is applied at the free end.

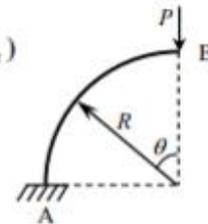
Vertical displacement at point B along the line of action of the load ( $\delta_{VB}$ )

$$\delta_{VB} = \frac{\pi PR^3}{4EI}$$

where,  $R$  = Radius of the quadrant,

$E$  = Young's modulus of the material of the beam

$$= 2 \times 10^5 \text{ N/mm}^2$$



- Moment of Inertia of the cross section of the curved member

$$I = \frac{bd^3}{12}$$

Horizontal displacement at point O (5.118)

$$\delta_{Ox} = \frac{PK}{2EI}$$

b. Quadrant of a circle with a straight leg.

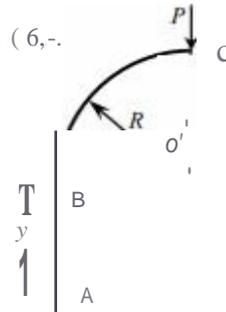
The member is a quadrant from point A to B and then straight line from B to C

Vertical displacement at point C along the line of action of the load (6.1)

$$\delta_{re} = \frac{nPR'y}{EI}$$

Horizontal displacement at point B (O.118)

$$\delta_{B,C} = -\frac{PR}{2EI} \left[ ry^2 + \frac{nR'}{8} + 4yR \right]$$



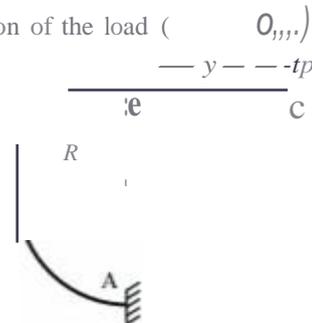
c. Semicircle with straight arm

Vertical displacement at point C along the line of action of the load (O.111)

$$\delta_{Cv} = \frac{PR}{EI} \left[ nR' + \frac{y^2}{2} \right]$$

Horizontal displacement at point B (O.118)

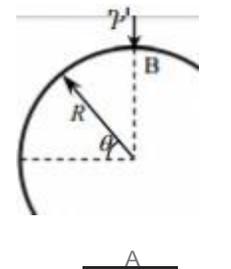
$$\delta_{BC} = -\frac{PR'}{EI} [nR + l_2]$$



d. Circle

Vertical displacement at point C along the line of action of the load (O.111)

$$\delta_{Cv} = -\frac{nPR'}{21EI}$$



Procedure:

- i. Place a load of 0.5 kg on the hanger to activate the member and treat this as the initial position for measuring deflection.
- ii. Fix the dial gauges for measuring horizontal and vertical deflections.
- iii. Place the additional loads at an increment of 0.5 kg and tabulate the dial gauge readings against the applied loads.

**Observation:***a. Quadrant*

S. No	Load kg	Vertical deflection (mm)				Horizontal deflection (mm)			
		Dial Gauge reading			Theoretical	Dial Gauge reading			Theoretical
		Initial	Final	Actual	$\delta_{VB}$	Initial	Final	Actual	$\delta_{HB}$
1	0.5								
2	1.0								
3	1.5								
4	2.0								

*b. Quadrant with straight leg*

S. No	Load kg	Vertical deflection (mm)				Horizontal deflection (mm)			
		Dial Gauge reading			Theoretical	Dial Gauge reading			Theoretical
		Initial	Final	Actual	$\delta_{VB}$	Initial	Final	Actual	$\delta_{HB}$
1	0.5								
2	1.0								
3	1.5								
4	2.0								

*c. Semicircle with straight arm*

S. No	Load kg	Vertical deflection (mm)				Horizontal deflection (mm)			
		Dial Gauge reading			Theoretical	Dial Gauge reading			Theoretical
		Initial	Final	Actual	$\delta_{VB}$	Initial	Final	Actual	$\delta_{HB}$
1	0.5								
2	1.0								
3	1.5								
4	2.0								

*d. Circle*

S. No	Load kg	Vertical deflection (mm)				Horizontal deflection (mm)			
		Dial Gauge reading			Theoretical	Dial Gauge reading			Theoretical
		Initial	Final	Actual	$\delta_{VB}$	Initial	Final	Actual	$\delta_{HB}$
1	1								
2	2								
3	3								
4	4								

*Dimensions of the beam*

S. No	Configuration	Quadrant	Quadrant with straight leg	Semicircle with straight arm	Circle
1	Main Scale				
	Vernier Scale				
	Total				
2	Radius				
3	Arm/leg length	-			-

**Calculation:**

**Precautions:**

- i. Apply the loads gently
- ii. Measure the displacements very accurately

**Results:**

1. Plot the graph between load and deflection for each case to show that the structure remains within the elastic limit.
2. Vertical deflection in mm,  $\delta_{VB}$

S. No	Case (a)		Case (b)		Case (c)		Case (d)	
	Exp.	Calc.	Exp.	Calc.	Exp.	Calc.	Exp.	Calc.
1								
2								
3								
4								

3. Horizontal deflection,  $\delta_{HB}$

S. No	Case (a)		Case (b)		Case (c)		Case (d)	
	Exp.	Calc.	Exp.	Calc.	Exp.	Calc.	Exp.	Calc.
1								
2								
3								
4								

**Comments:**

## Experiment:- 8

### Analysis of Redundant frame

#### Aim:

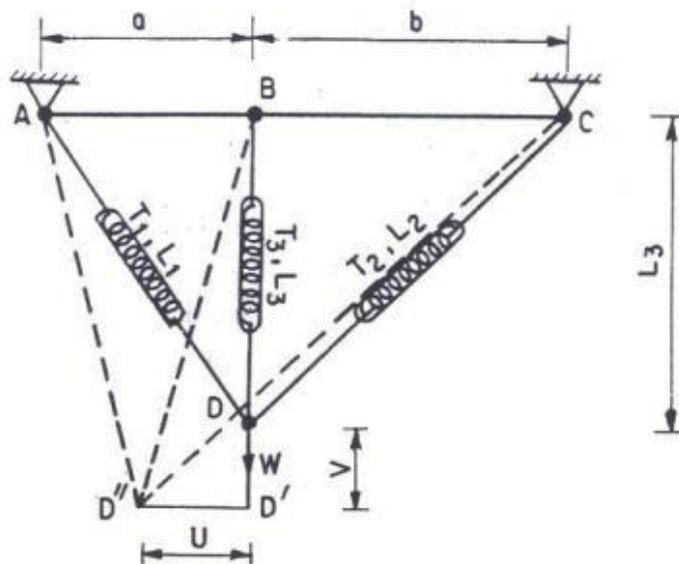
To study the behaviour redundant frame subjected to coplanar force experimentally and to verify the horizontal and vertical displacements obtained from the experiment with the analytical results.

#### Apparatus:

Three bar suspension system, weights, scale, dial gauge, etc.

#### Theory:

The diagram of the apparatus is shown in the figure below.



The horizontal ( $U$ ) and the vertical ( $V$ ) displacements of the point D is calculated as follows

$$U = \frac{W}{L_3} \times \frac{(N_1 a - N_2 b)}{N_1 N_2 (a+b)^2 + N_3 (n_1 a^2 + n_2 b^2)}$$
$$V = \frac{W}{L_3^2} \times \frac{(N_1 a^2 + N_2 b^2)}{N_1 N_2 (a+b)^2 + N_3 (n_1 a^2 + n_2 b^2)}$$

where,

$$N_1 = \frac{A_1 E_1}{L_1} \times \frac{1}{L_1^2}$$

$$N_2 = \frac{A_2 E_2}{L_2} \times \frac{1}{L_2^2}$$

$$N_3 = \frac{A_3 E_3}{L_3} \times \frac{1}{L_3^2}$$

$L_1$  = Length of the member AD

$L_2$  = Length of the member BD

$L_3$  = Length of the member CD

$a$  = Distance between A and B

$b$  = Distance between A and B

$W$  = Applied load at D

The tensile force in the members are calculated as follows

$$T_1 = \frac{(L_3 V - a U) A_1 E_1}{L_1^2}$$

$$T_2 = \frac{(L_3 V + b U) A_2 E_2}{L_2^2}$$

$$T_3 = \frac{(L_3 V) A_3 E_3}{L_3^2}$$

where,

$T_1$  = Tension force in member AD

$T_2$  = Tension force in member BD

$T_3$  = Tension force in member CD

The expression  $\frac{AE}{L}$  represents the axial stiffness of the structure. It denotes the force required to produce unit deformation. This value can be calculated by finding the slope from load vs. deflection graph plotted for each spring.

### Procedure:

1. Isolate each spring, apply load and measure the deflection and tabulate it.
2. Draw a graph between load ( $y$  - axis) and deflection ( $x$  - axis) for each spring and find the slope. The value of the slope corresponds to the stiffness of each spring.
3. Connect the lower end of the spring to make a redundant frame.

4. Apply load at increments and note down the horizontal and vertical displacements and the reading in each spring.
5. Calculate the tension force in each spring, horizontal and vertical displacement of point D and compare with the experimental results.

**Observation:**

Length of member AD =  
 Length of member BO =  
 Length of member CD =  
 Distance  $a$  =  
 Distance  $b$  =  
 Young's Modulus,  $E$  =

Load, kg	Deflection in member, mm		
	AD	BO	CD
1.0			
2.0			
3.0			
4.0			

load, kg	Deflection, mm				
	Horizontal, $U$	Vertical, $V$	Spring AD	Spring BO	Spring CD
1.0					
2.0					
3.0					
4.0					

**Calculation:**

**Precaution:**

- i. Calculate the spring stiffnesses carefully.
- ii. Measure the distances AD, BD, CD,  $a$  and  $b$  accurately.
- iii. Tap the dial gauges before taking a reading for vertical and horizontal displacements.

**Results:**

Load, kg	Experimental					Analytical				
	Deflection, mm		Force, N			Deflection, mm		Force, N		
	$U$	$V$	$T_1$	$T_2$	$T_3$	$U$	$V$	$T_1$	$T_2$	$T_3$
1.0										
2.0										
3.0										
4.0										

**Comments:**

