Modern Physics: Classical to Quantum Physics

Prof. Prince A Ganai

NIT-Srinagar INDIA

May 2025

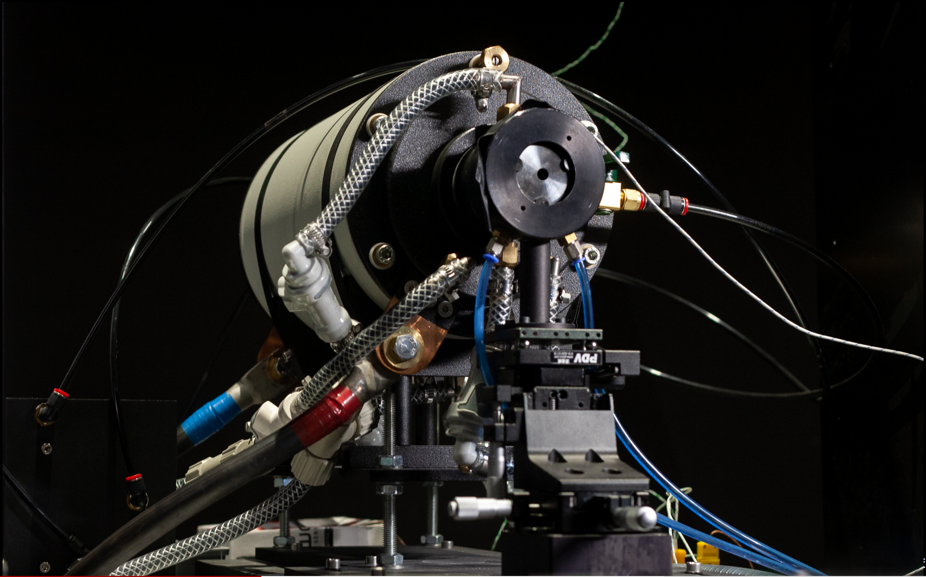
Outline

What is Blackbody Radiation?

- A **blackbody** is an idealized object that absorbs all incident radiation.
- It also emits radiation depending on temperature.
- Emission is **continuous and temperature dependent**.

Example: heated cavity with small hole

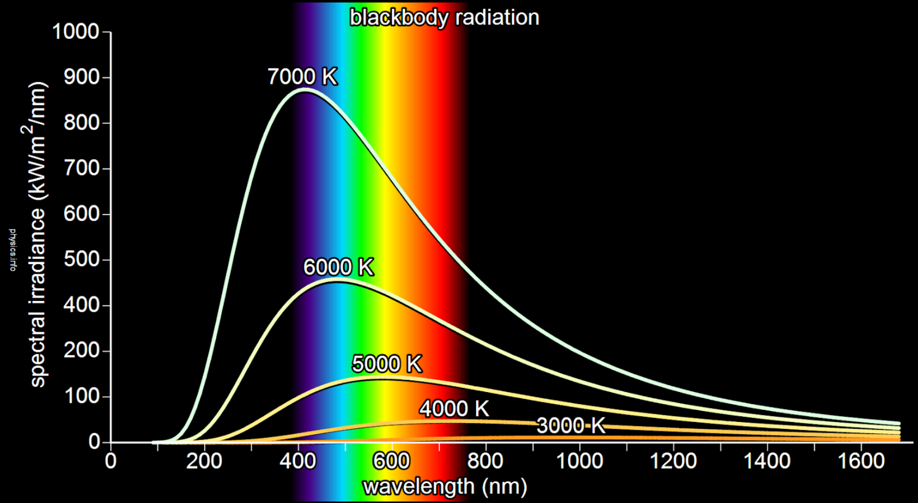
Observed Spectral Distribution



Importance in Physics

- Early 20th-century physics couldn't explain the spectrum.
- Led to the **birth of quantum mechanics**.
- Blackbody radiation affects:
  - Thermodynamics
  - Astrophysics
  - Quantum theory

Observed Spectral Distribution



- Curve shifts with temperature. Has a distinct peak.

Wien's Displacement Law

$$\lambda_{\text{max}} T = \text{constant} = 2.898 \times 10^{-3} \text{ m·K}$$

As temperature increases:

- Peak shifts left (shorter wavelengths)
- Total emitted power increases

Rayleigh-Jeans Law (Classical)

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} kT$$

$$\Rightarrow \text{As } \nu \to \infty, \quad u(\nu, T) \to \infty$$

$\rightarrow$ **Ultraviolet Catastrophe!**

Ultraviolet Catastrophe

- Classical physics predicted **infinite energy** at high frequencies.
- Total energy diverges:

$$U = \int_0^\infty u(\nu, T) d\nu \to \infty$$

Contradicts experiment!

Why Classical Physics Failed

- Treats energy as continuous.
- Assumes equal probability of all modes.
- Ignores quantum nature of energy exchange.

A new hypothesis was needed.

Objective

- Derive Planck's law of blackbody radiation:

$$u(\nu, T) = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{e^{h\nu/kT} - 1}$$

- Approach:
  1. Count EM modes in a cavity
  2. Assign quantized energy levels
  3. Use Boltzmann statistics to find average energy

Standing Waves in a Cubic Cavity

- Consider a cube of side L
- Boundary condition: standing waves
- Allowed wavevectors:
$$k_x = \frac{\pi n_x}{L}, \quad k_y = \frac{\pi n_y}{L}, \quad k_z = \frac{\pi n_z}{L}$$
- $n_x, n_y, n_z \in \mathbb{Z}^+$

Total Number of Modes

- Wavenumber $k = \sqrt{k_x^2 + k_y^2 + k_z^2}$
- Each mode occupies a point in 3D k-space
- Volume of shell in k-space between k and $k + dk$:

$$dN_k = \frac{1}{8} \cdot \frac{4\pi k^2 dk}{\left(\frac{\pi}{L}\right)^3}$$

$$dN_k = \frac{V}{2\pi^2} k^2 dk, \quad V = L^3$$

Convert to Frequency Domain

$$\nu = \frac{c}{2\pi}k \quad \Rightarrow \quad k = \frac{2\pi\nu}{c}, \quad dk = \frac{2\pi}{c}d\nu$$

Substitute into dN_k:

$$dN_\nu = \frac{8\pi V\nu^2}{c^3}d\nu$$

Each mode has 2 polarizations $\rightarrow$ multiply by 2:

$$g(\nu)d\nu = \frac{8\pi V\nu^2}{c^3}d\nu$$

Planck's Quantum Hypothesis

- Energy of oscillator at frequency ν is quantized:

$$E_n = nh\nu, \quad n = 0, 1, 2, \ldots$$

- Only integer multiples of $h\nu$ are allowed
- Use this with statistical mechanics

Average Energy of an Oscillator

$$\langle E \rangle = \frac{\sum_{n=0}^{\infty} E_n e^{-E_n/kT}}{\sum_{n=0}^{\infty} e^{-E_n/kT}} = \frac{\sum_{n=0}^{\infty} nh\nu e^{-nh\nu/kT}}{\sum_{n=0}^{\infty} e^{-nh\nu/kT}}$$

Use identity:

$$\sum_{n=0}^{\infty} ne^{-nx} = \frac{e^{-x}}{(1-e^{-x})^2} \quad \text{and} \quad \sum_{n=0}^{\infty} e^{-nx} = \frac{1}{1-e^{-x}}$$

Set $x = \frac{h\nu}{kT}$

Result: Average Energy Per Mode

$$\langle E \rangle = \frac{h\nu}{e^{h\nu/kT} - 1}$$

This is the **Planck distribution** for one oscillator at frequency ν

Energy Density per Unit Frequency

$$u(\nu, T)d\nu = \frac{\text{Total energy in modes between } \nu \text{ and } \nu + d\nu}{\text{Volume}}$$

$$= \frac{g(\nu)}{V} \cdot \langle E \rangle = \frac{8\pi\nu^2}{c^3} \cdot \frac{h\nu}{e^{h\nu/kT} - 1}$$

$$\boxed{u(\nu, T) = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{e^{h\nu/kT} - 1}}$$

Energy Density per Wavelength (optional)

Use:

$$\nu = \frac{c}{\lambda}, \quad d\nu = -\frac{c}{\lambda^2} d\lambda$$

$$u(\lambda, T) = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1}$$

(Planck's Law in terms of λ)

Key Results Recap

- Number of modes per frequency: $\frac{8\pi\nu^2}{c^3}$
- Energy quantized: $E_n = nh\nu$
- Average energy: $\langle E \rangle = \frac{h\nu}{e^{h\nu/kT}-1}$
- Final result:

$$u(\nu, T) = \frac{8\pi h\nu^3}{c^3} \cdot \frac{1}{e^{h\nu/kT} - 1}$$

Why It Matters

- Resolved the ultraviolet catastrophe.
- Introduced the quantum of action: h
- Opened the door to quantum mechanics.

Planck's law was the first step into the quantum world.

Thank You!

Questions and Discussion