Modern Physics: Classical to Quantum Physics
Compton Effect

Prof. Prince A Ganai

NIT-Srinagar INDIA

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Outline

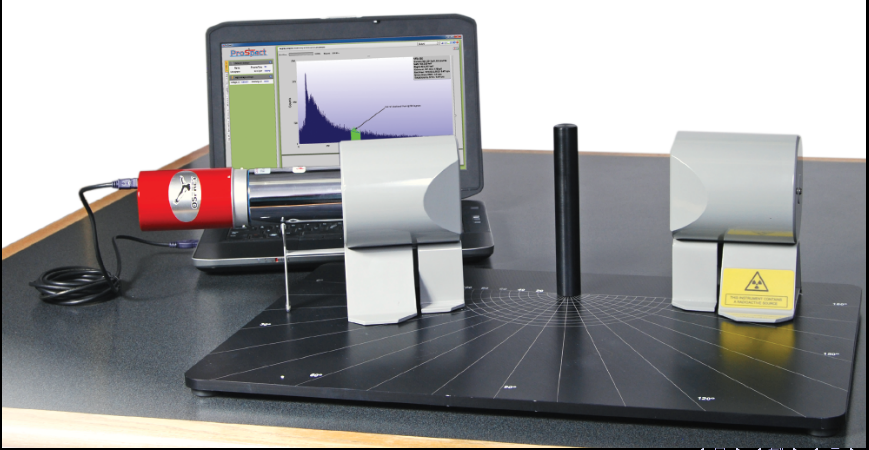
Motivation: Classical Picture Fails

- X-rays scattered by electrons show wavelength change!
- Classical EM theory predicts no such change.
- This was unexplained until **Compton (1923)** proposed a quantum model.

$\longrightarrow$ Needed a model treating light as photons with momentum.

Compton's Experiment

- Incident monochromatic X-rays scatter off electrons in graphite.
- Observed: scattered X-rays have longer wavelength ($\lambda' > \lambda$).



Key Observation

- Wavelength shift $\Delta\lambda = \lambda' - \lambda$ depends only on scattering angle.
- Independent of intensity and target material (provided electrons are free).

$$\boxed{\Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta)}$$

This is the **Compton shift formula**.

Photon Energy and Momentum

$$E_\gamma = h\nu = \frac{hc}{\lambda}, \quad p_\gamma = \frac{E}{c} = \frac{h}{\lambda}$$

$\Rightarrow$ Photons carry momentum, despite having zero rest mass!

Electron Energy and Momentum

$$E_e^2 = p_e^2 c^2 + m_e^2 c^4$$

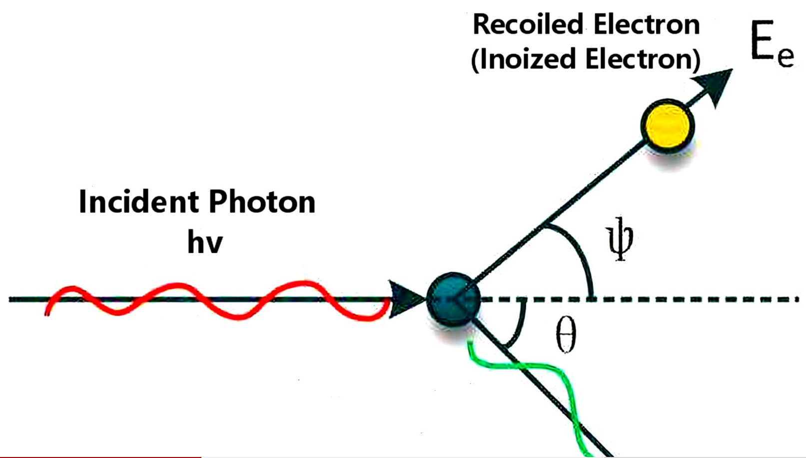
Initially: electron is at rest

$$E_{\text{initial}} = m_e c^2$$

After collision: gains kinetic energy and momentum

Collision Geometry

- Incident photon: energy $E = h\nu$, wavelength λ
- Scattered photon: angle θ, energy $E' = h\nu'$, wavelength λ'
- Recoil electron: angle ϕ



Conservation Laws

- Apply:

$$\text{(1) Energy conservation:} \quad h\nu + m_e c^2 = h\nu' + E_e$$

- And:

$$\text{(2) Momentum conservation (vector) :} \quad \vec{p}_\gamma = \vec{p}'_\gamma + \vec{p}_e$$

Step 1: Use 4-Momentum

Use relativistic 4-momentum conservation: $P_\gamma + P_e = P'_\gamma + P'_e$

Start with energy:

$$h\nu + m_e c^2 = h\nu' + \sqrt{(p_e c)^2 + (m_e c^2)^2}$$

Step 2: Square Energy Equation

$$\left(h\nu + m_e c^2 - h\nu'\right)^2 = p_e^2 c^2 + m_e^2 c^4$$

Also from momentum conservation (x and y components):

$$\vec{p}_\gamma - \vec{p}'_\gamma = \vec{p}_e$$

Step 3: Use Dot Product

$$p_e^2 = p_\gamma^2 + p_\gamma'^2 - 2 p_\gamma p_\gamma' \cos\theta$$

Substitute: $p_\gamma = \frac{h}{\lambda}$, $p_\gamma' = \frac{h}{\lambda'}$

Step 4: Eliminate Electron Momentum

After algebraic manipulation:

$$\frac{1}{\lambda'} - \frac{1}{\lambda} = \frac{h}{m_e c}(1 - \cos\theta)$$

Invert and rearrange:

$$\boxed{\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)}$$

The Compton Wavelength

$$\lambda_C = \frac{h}{m_e c} = 2.43 \times 10^{-12}\ \text{m}$$

$$\Delta\lambda = \lambda_C(1 - \cos\theta)$$

Why Was This Revolutionary?

- Confirmed that light behaves like particles (photons).
- Conservation of energy + momentum applies to light-matter interaction.
- Classical wave theory fails completely to predict wavelength shift.

Particle-Wave Duality

- Compton effect supports:
  - Particle nature of light (photon momentum)
  - Wave-particle duality: light and matter both have dual character

Links with:

- Photoelectric effect
- de Broglie hypothesis
- Quantum mechanics foundation

Angular Dependence

$$\Delta\lambda \propto (1 - \cos\theta)$$

- Maximum shift at $\theta = 180^\circ$
- Zero shift at $\theta = 0^\circ$

Key Equations Recap

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

$$\lambda_C = \frac{h}{m_e c} \approx 2.43 \times 10^{-12}\ \text{m}$$

$$E = h\nu, \quad p = \frac{h}{\lambda}, \quad E^2 = p^2c^2 + m^2c^4$$

Conceptual Summary

- Compton effect demonstrates photon momentum.
- Relativity and quantum ideas work together.
- Essential experiment in the development of quantum physics.

Classical theory dies, quantum theory begins.

Thank You!

Questions and Discussion

Questions 1–2

Q1. Derivation of Compton Shift

Derive the Compton shift formula:

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$

Hint: Use conservation of energy and momentum.

Q2. Maximum Wavelength Shift

Calculate the maximum possible change in wavelength for $\theta = 180^\circ$. Use $\frac{h}{m_e c} = 0.00243\,\text{nm}$.

Questions 3–4

Q3. Photon Energy Change

A photon of 0.1 nm wavelength is scattered at 90°. Calculate:

- (a) Wavelength shift
- (b) Energy lost by photon
- (c) Energy gained by electron

Q4. Finding Scattering Angle

A photon's wavelength increases by 0.001 nm. Find the scattering angle.

Questions 5–6

Q5. Recoiling Electron Kinetics

A photon of $100\,\mathrm{keV}$ is scattered at 120°. Calculate:

- (a) Energy of scattered photon
- (b) Kinetic energy of electron
- (c) Recoil angle (qualitative)

Q6. Visible Light and Compton Effect

Explain why Compton scattering is not observed for visible light.

Q7. Compton Wavelength of Electron

Define and calculate the Compton wavelength of the electron. Discuss its significance.

Q8. Energy Transfer Efficiency

A 1 MeV photon scatters off a stationary electron. What is the maximum fraction of energy it can transfer to the electron?

Questions 9–10

Q9. Backscattered Photon

An X-ray photon of 200 keV is backscattered. Calculate:

- (a) Energy of scattered photon
- (b) Kinetic energy of electron
- (c) Momentum of electron

Q10. Experiment vs Theory

Incident wavelength: 0.07 nm, scattered: 0.075 nm. Estimate the scattering angle and compare with theory.

References

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