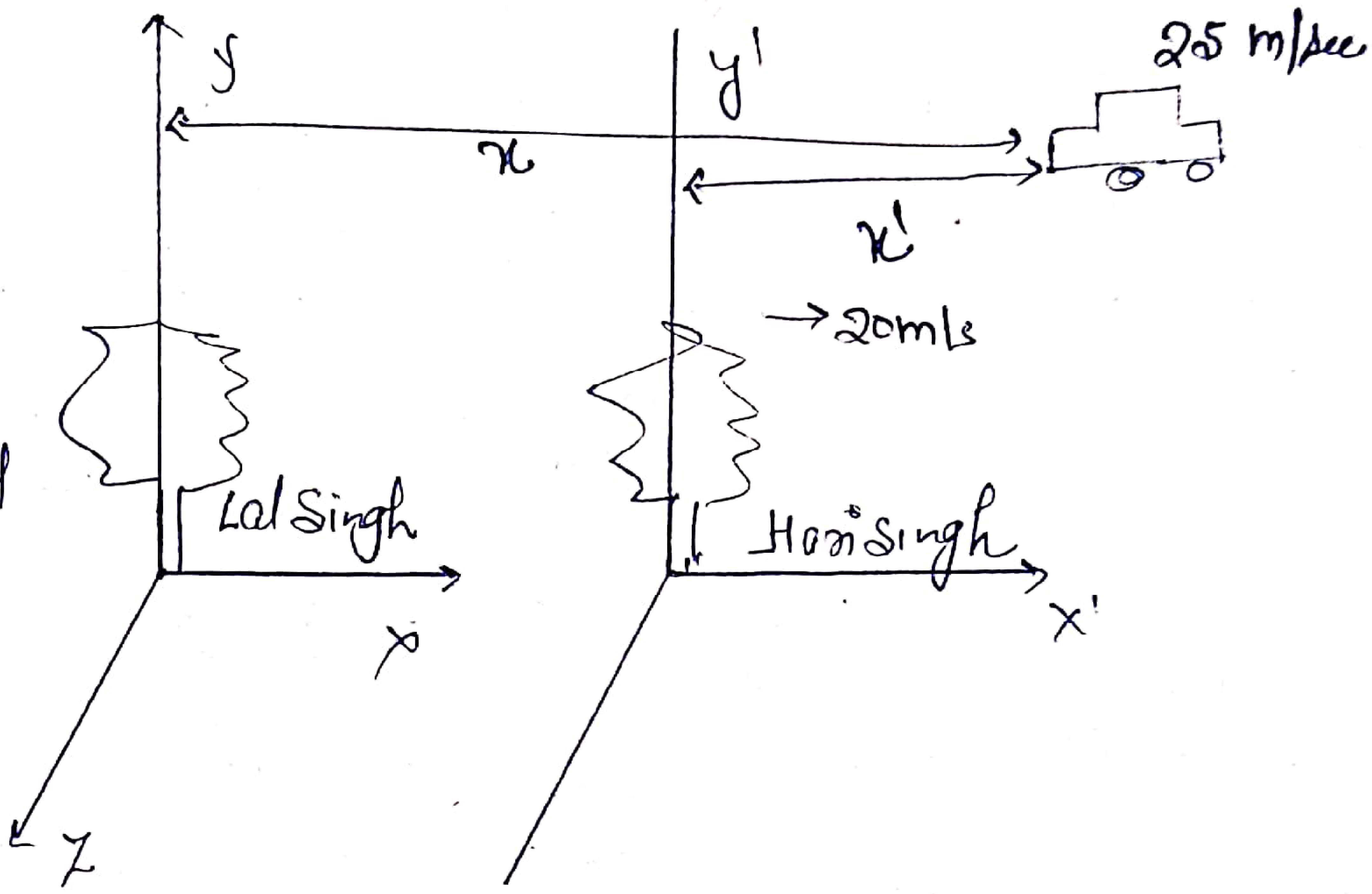
Introduction to theory of Relativity:-

Lal Singh and Hari Sing are one sitting under different trees. Both are fully equipped with the physical instrument.



CASE-I Both are Rest

Quantity	Lal Singh	Hari Singh	opinion
Position of Car	x	x'	Dis Agree
displacement	ΔD (5m)	ΔD(5m)	Agree
Size of Car	l	l	Agree
Mass of Car	m	m	Agree
Velocity of Car	v (25)	v (25)	Agree
Eat pizza/Burger	Δt	Δt	Agree
Acc^n	a	a	Agree

$F = ma$

Newton's law obeyed

CASE-II Lal Singh at Rest Hari moves with Constant Velocity (20m/s)

Quantity	Lal	Hari	opinion
Postion of Car	x	x'	DisAgree
displacement	ΔD	ΔD	Agree
Size of Car.	l	l	Agree
Mass of Car	m	m	Agree
Velocity of Car	v(25)	v'(5)	disAgree
Eat pizza/Burger	Δt	Δt	Agree.
Acc^n	a	a	Agree

CASE-III

Lal Singh at Rest, Hari Singh moves with Constant Velocity (2.5×10^8 m/s) [Experimentally proved]

Quantity	Lal Singh	Hari Singh	opinion
Postion of Car.	x	x'	Disagree
displacement	ΔD	$\Delta D'$	disagree.
Size of Car	l	l'	dis Agree
Mass of Car.	m	m'	Disagree
Velocity of Car	v	v'	Disagree
Eat Pizza/Burger	Δt	$\Delta t'$	Disagree
Acc^n	a	a'	disagree

Newton's law not obeyed.

Newtonian OR Galilean Theory of Relativity

CASE-I, CASE-II — Theory of Relativity

CASE-III → Special Theory of Relativity Gives by Elbert Einestein

CASE-IV. If Hari is Acc^n

General theory of Relativity.

Special Theory of Relativity:-

Frame of Referance, which are moving with Constant velocity

General Theory of Relativity:-

Accⁿ Frame of Referance ~~moving with~~

For Case I & Case II

Space and time are absolute

Case III

Space and time are not ~~abs~~ absolute.

Basic Terms of Relativity:-

(i) Event:- Any kind of happening is called event.

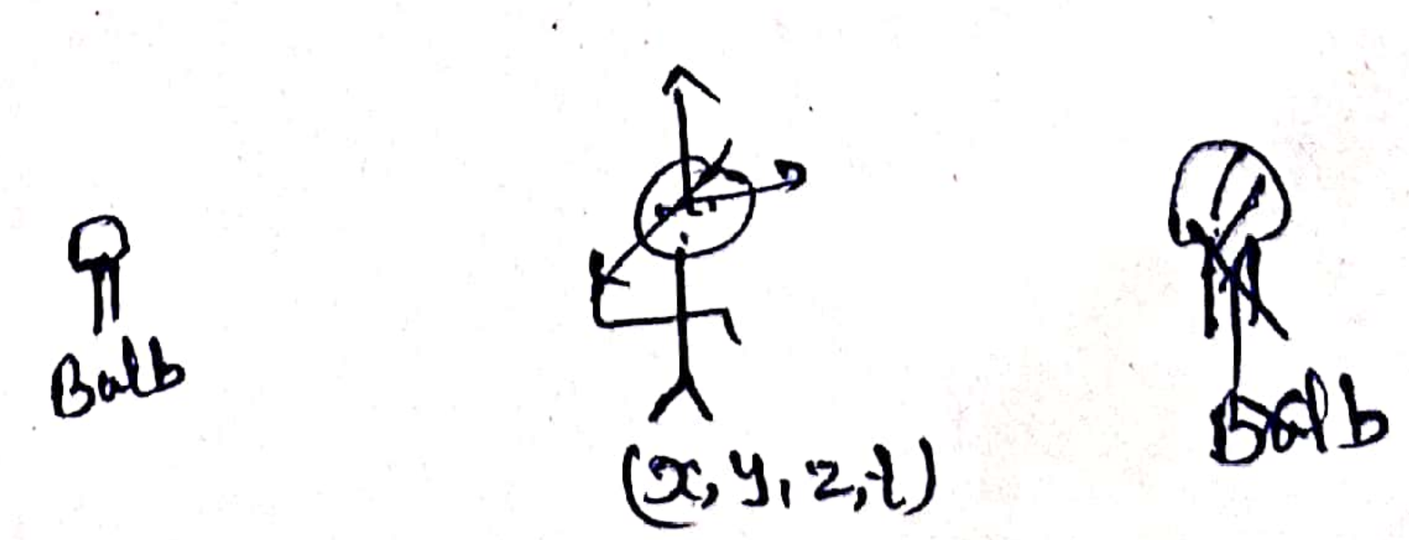
Blinking the eyes, ON-OFF of Bulb Switch

event does not depend on observer/

event is independent of referance frame

event → Space ~~When~~ where
→ Time when

event is expressed as (Space-time) (x, y, z, t)

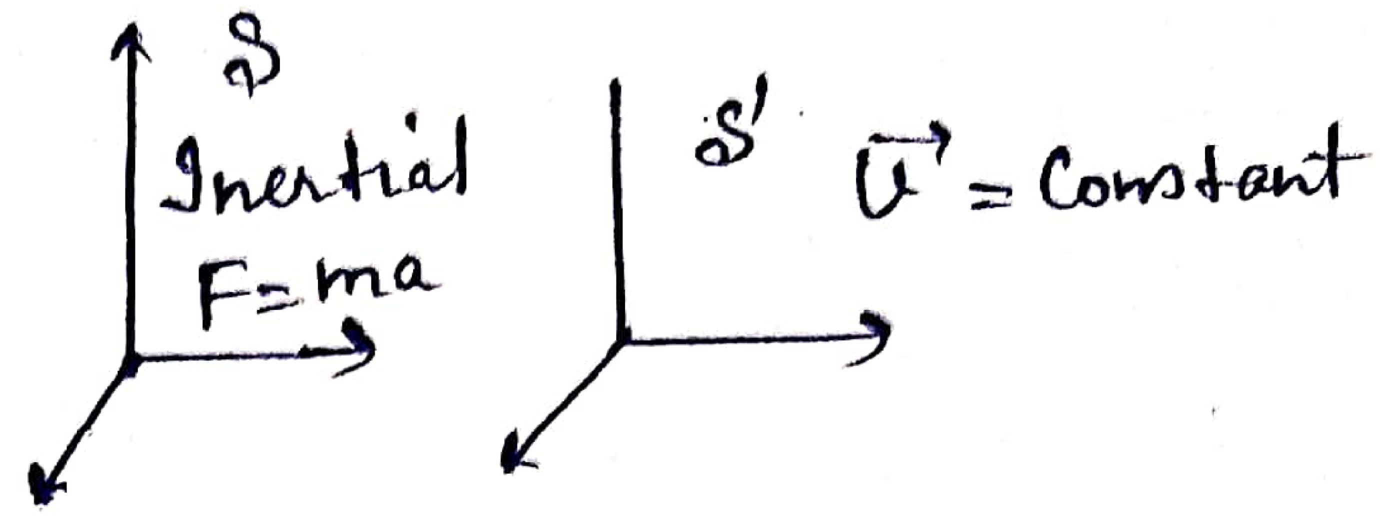


Frame of Referenes:-

Any Coordinate System which is select for any measurement known as frame of referance..

Co-ordinate system, time → frame of referance

Inertial frame of Reference:-



If $F=0$
No force
$\vec{v}$ = Constant
Particles moving with const. speed in a ~~straight~~ straight path

Newton's law obey.

$$\boxed{\vec{F} = m\vec{a}}$$

An inertial frame of reference is one in which Newton's first law of motion holds. In such a frame, an object at rest remains at rest and an object in motion continues to move at constant velocity (Constant speed and direction) if no force acts on it.

Any frame of reference that moves at constant velocity relative to an intertial frame is it self an ~~int~~-inertial frame.

Distant star, → Inertial frame of Reference

Observer :→ Fully equiped, imaginary Instrument

Invariant Quantity :→ Quantity which does not vary from one frame reference to another frame of refernce

Charge,

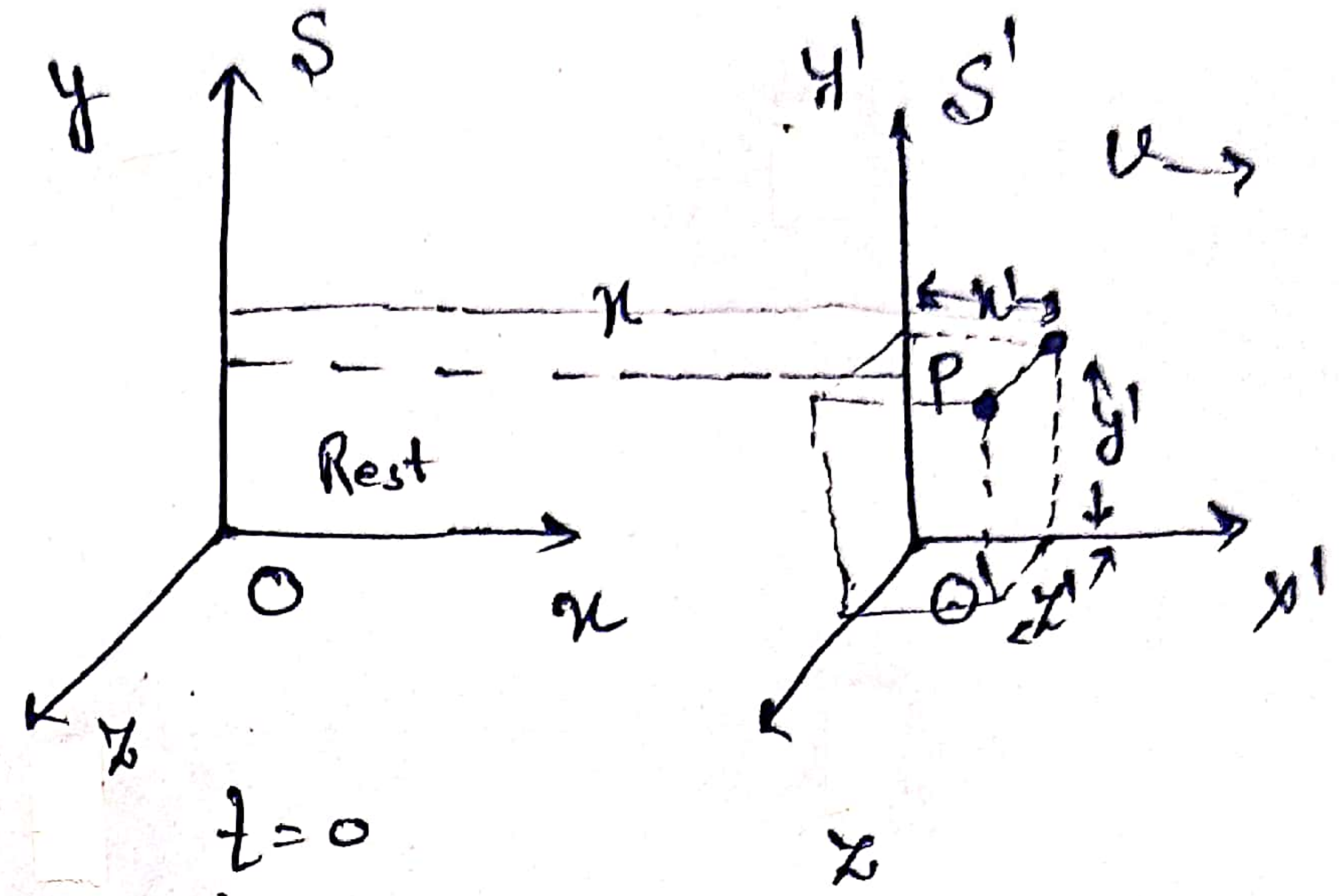
Velocity → Variant Quantity

absolute Quantity → No variation (In variant)

Earth mass,

Galilean Transformation :-

One frame of reference is indicated by S at Rest.
Second frame of Refence S' is moving with the Constant Velocity $\vec{v}$.



$$x = x' + vt \Rightarrow x' = x - vt$$

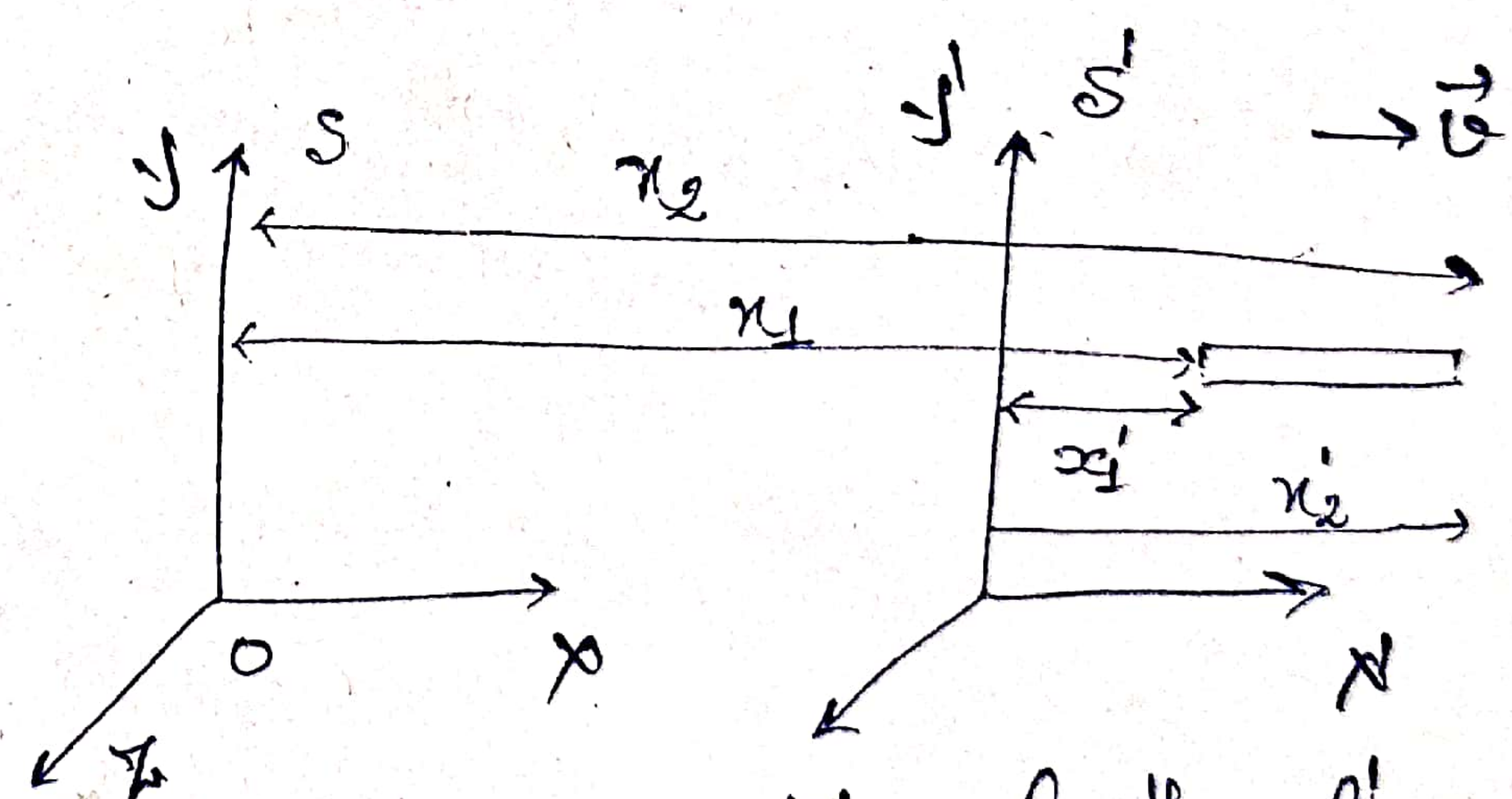
$$y' = y \qquad z' = z$$

$t = t'$ According to Newton's (Classical) time is absolute : Donot change with motion of observer"

Time interval is same for every inertial frame of reference.

Galilean Invariance :→

- Galilean transformation



$$x' = x - vt$$

$$x_2' = x_2 - vt$$

$$x_1' = x_1 - vt$$

$l = x_2 - x_1$

Length $= l'$

$l' = x_2' - x_1'$

$$l' = x_2' - x_1' = x_2 - vt - (x_1 - vt)$$

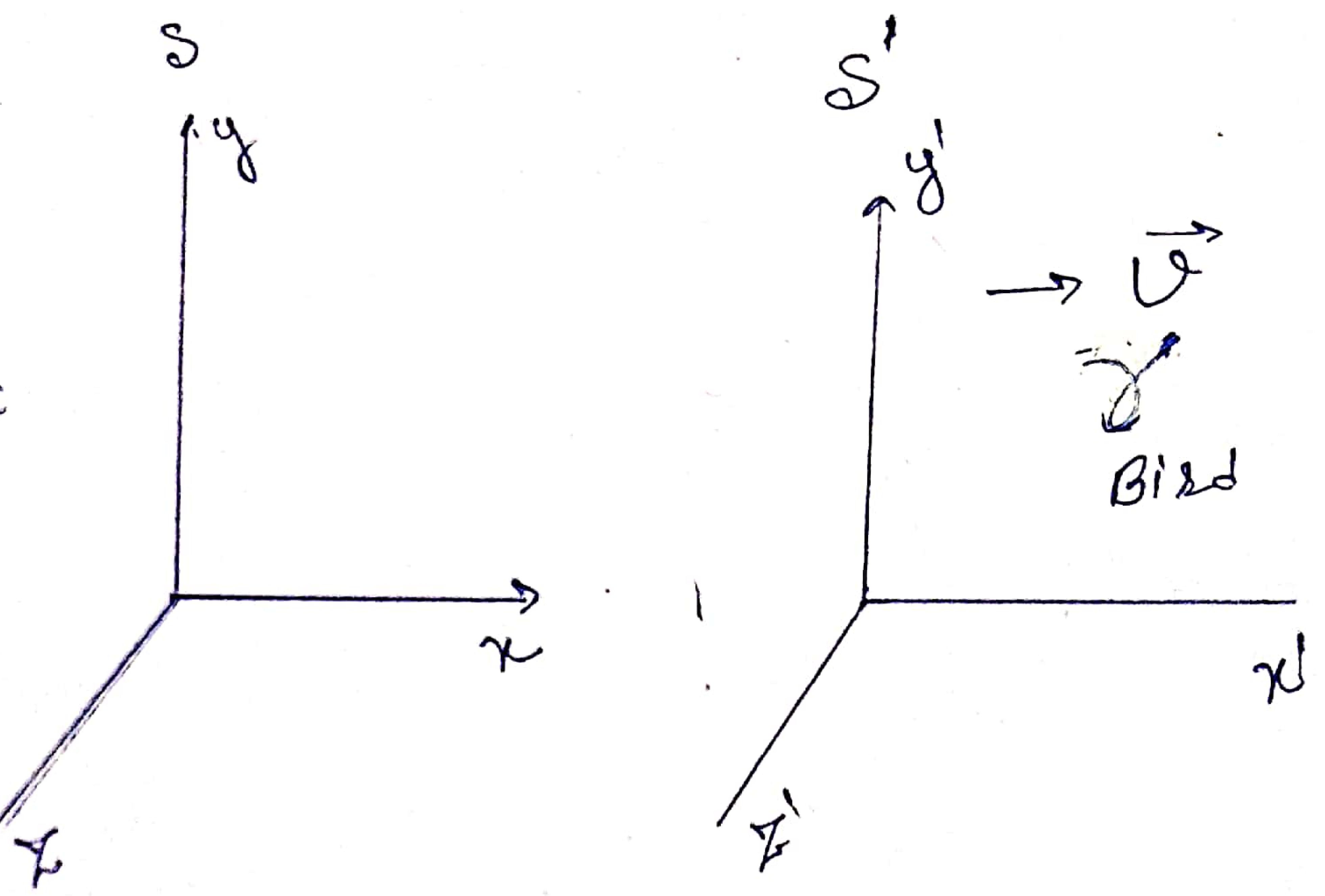
$$l' = x_2 - x_1$$

$\boxed{l' = l}$ Length is invarient under ~~Gal~~ Galilean transformation
Space is absolute in Galilean transformation

Space is absolute in Galilean transformation

$$\left.\begin{array}{l}\text{Length is absolute}\\ \text{time is absolute}\\ \text{Mass is absolute}\end{array}\right] \rightarrow \text{Newtonian Relativity}$$

Velocity



$\vec{u} = u_x \hat{i} + u_y \hat{j} + u_z \hat{k}$

$\vec{u'} = u'_x \hat{i} + u'_y \hat{j} + u'_z \hat{k}$

$x' = x - vt$, $y' = y$, $z' = z$, $t' = t$

$\dfrac{dx'}{dt} = \dfrac{dx}{dt} - v$ | $\dfrac{dy'}{dt} = \dfrac{dy}{dt}$ | $\dfrac{dz'}{dt} = \dfrac{dz}{dt}$

$u'_x = u_x - v$ | $u'_y = u_y$ | $u'_z = u_z$

In term of Vector

$$u'_x \hat{i} + u'_y \hat{j} + u'_z \hat{k} = u_x \hat{i} + u_y \hat{j} + u_z \hat{k} - \vec{v}$$

$$\boxed{\vec{u'} = \vec{u} - \vec{v}}$$

Velocity is Variant quantity.

Again differential

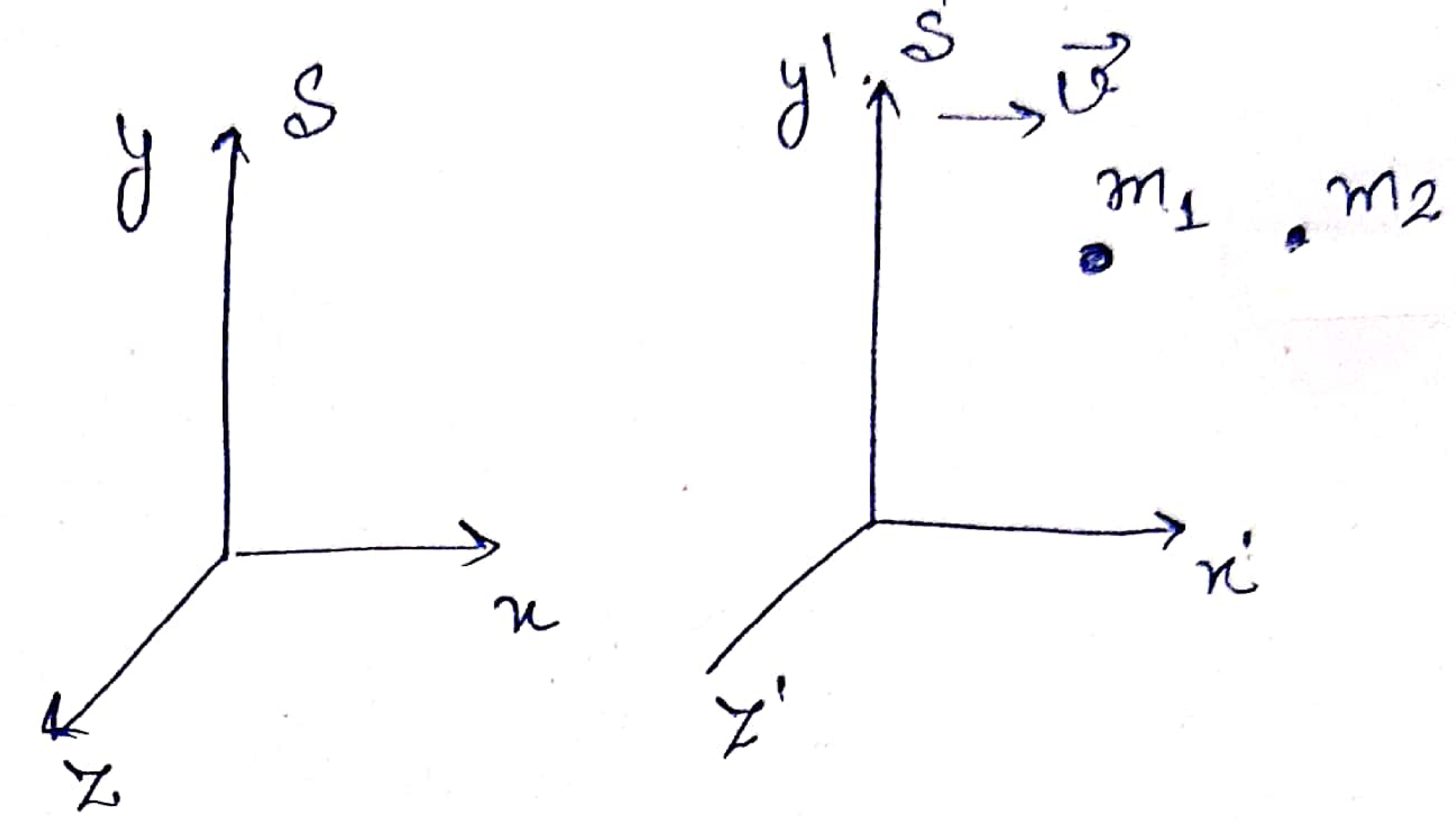
$$\frac{d^2x'}{dt^2} = \frac{d^2x}{dt^2}, \quad \frac{d^2y'}{dt^2} = \frac{d^2y}{dt^2}, \quad \frac{d^2z'}{dt^2} = \frac{d^2z}{dt^2}$$

$a'_x = a_x$ | $a'_y = a_y$ | $a'_z = a_z$

$$a'_x \hat{i} + a'_y \hat{j} + a'_z \hat{k} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$$

$\boxed{a' = a}$ Acceleration is invariant.

Conservation of Linear Momentum:



$u_1 \rightarrow$ Velocity of first object before Collision for frame S

$u_2 \rightarrow$ " Second " " " " S

$v_1 \rightarrow$ " " first object after " " " S

$v_2 \rightarrow$ " " Second object after " " S

According to law of Conservation of Linear momentum

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \quad \text{— (6.1)}$$

From Galilean transformation

$$\vec{u'} = u - v$$

u_1' = Velocity of Ist object before Collision in frame S'

u_2' = " " IInd " " " S'

v_1' = " " Ist object after " " S'

v_2' = " " IInd object after " " S'

$$\boxed{u_1' = u_1 - v} \quad \boxed{u_2' = u_2 - v}$$

$$\boxed{u_1 = u_1' + v} \quad \boxed{u_2 = u_2' + v}$$

$$\boxed{v_1' = v_1 - v} \quad \boxed{v_2' = v_2 - v}$$

$$\boxed{v_1 = v_1' + v} \quad \boxed{v_2 = v_2' + v}$$

(6.2)

$$m_1(u_1' + v) + m_2(u_2' + v) = m_1(v_1' + v) + m_2(v_2' + v)$$

$$m_1 u_1' + m_2 u_2' = m_1 v_1' + m_2 v_2' \quad \text{— (7.1)}$$

Linear momentum before Collision in S' — Linear momentum after Collision in S'.

Law of Conservation of linear momentum as invariant under Galilean transformation

Conservation of Energy

From Galilean transformation

$$\vec{u}' = \vec{u} - \vec{v}$$

for frame S

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\frac{1}{2} m_1 (v + u_1')^2 + \frac{1}{2} m (v + u_2')^2 = \frac{1}{2} m_1 (v + v_1')^2 + \frac{1}{2} m_2 (v + v_2')^2$$

On solving

$$\frac{1}{2} m_1 u_1'^2 + \frac{1}{2} m_2 u_2'^2 + \underline{v(m_1 u_1' + m_2 u_2')} = \frac{1}{2} m v_1'^2 + \frac{1}{2} m v_2'^2 + \underline{v[m_1 v_1' + m_2 v_2']}$$

From Conservation of linear momentum in S' frame

$$m_1 u_1' + m_2 u_2' = m_1 v_1' + m_2 v_2'$$

$$\boxed{\frac{1}{2} m_1 u_1'^2 + \frac{1}{2} m_2 u_2'^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m v_2'^2}$$

K.E before Collision = After Collision

Conservation of energy is also valid for both the frame.

K.E.

Mechanical energy → K.E. or P.E

K.E + P.E.

Ether Hypothesis :→

Why we need ether or medium

Sound wave , Water wave , String wave, earth quake

Medium → Air

medium → water

medium → string

* Every wave needs a medium for their propagation

wave - Transfer → Transfer of energy and momentum from one place to another place.

energy & momentum can not transfer without the help of medium.

Medium (Air) not travelling only disturbance is travelling.

* Energy and momentum transfer needs some medium

wave → Disturbance in something

Disturbance can not travel from one place to other place with out medium

* Light a wave which must require a medium: ETHER AETHER

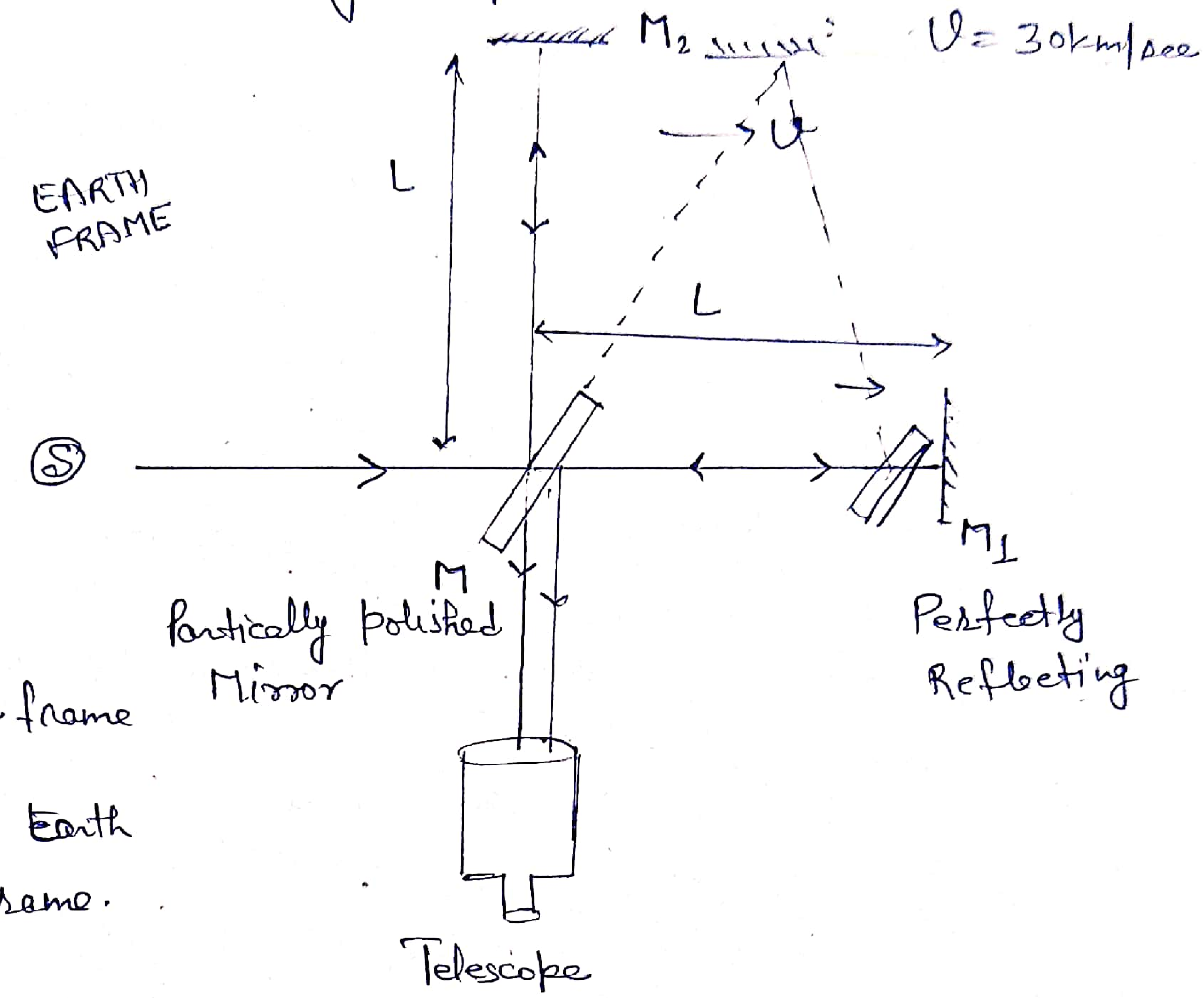
Properties of Ether

1. It prevades (spread) through all universe.
2. It is Infinitely high transparent.
3. It has very - very high density.
4. It is highly elastic.
5. It must be solid.

Michelson - Morley experiment

Speed of Earth relative to ether

Ether at Rest
Compare to sun



Interferometer

C → Speed of light in Ether frame

u → Speed of the Earth in Ether frame.

No phase difference after Reflection & transmission

Aim Whether the Velocity of light measured relative to the Earth remains same or Changes.

If direction change

340

100

340 − 100 = 240 km/sc

340 + 100 = 440 km/s

340 km/s 340 km/s

100 km/s

Relative Speed = 340 − 100 = 240 km/s

In opposite direction

Relative speed = 340 + 100 = 440 km/sec

Michelson – Morley Experiment

If velocity is less, ray cover less path

If Velocity is more ray cover more path

If observer move with 100 km/s in forward direction

Observer feels speed = 340 − 100 [c−v]

Relative speed = 240 km/s

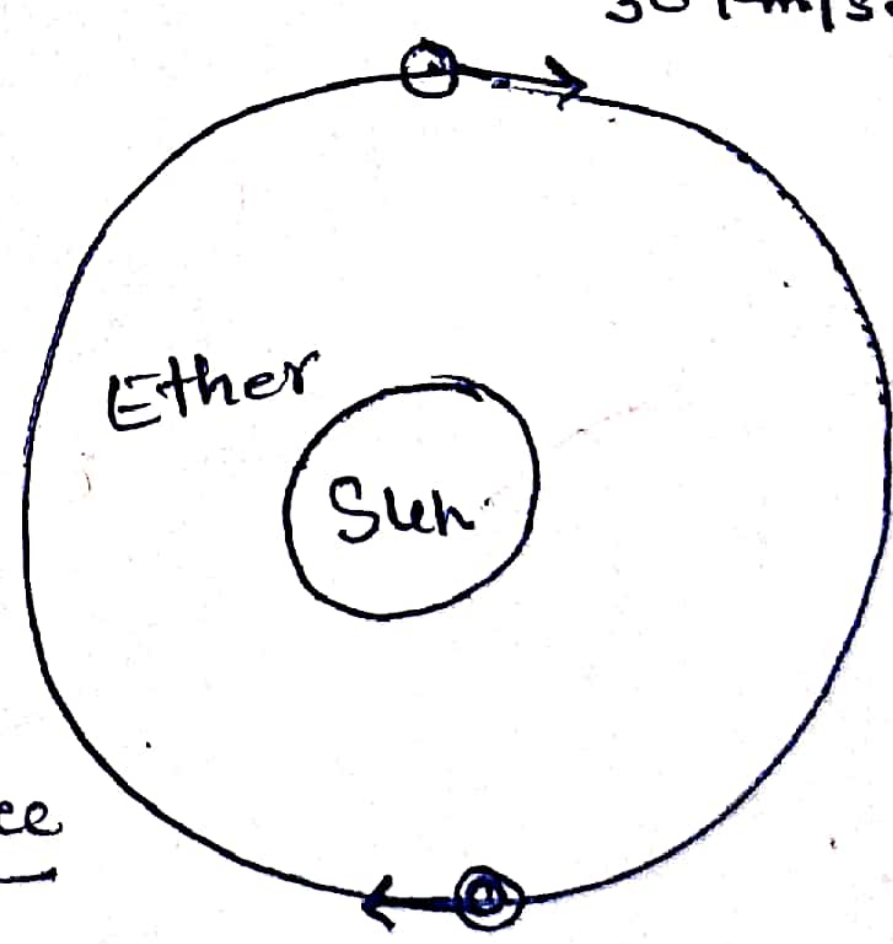
If observer feel the speed in opposite direction = 340 + 100

= 440 km/s [c+v]

Conclusion:

If any observer is moves, the relative speed will change

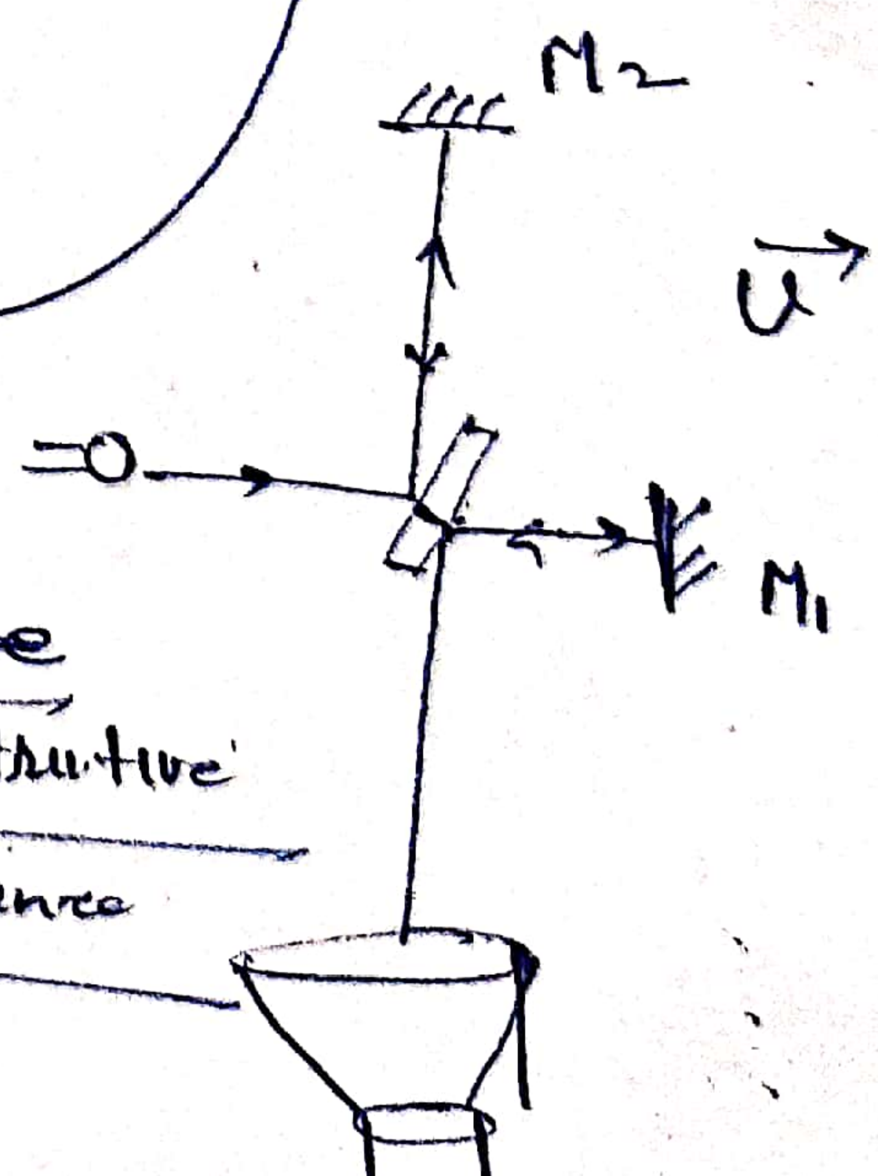
Ether is present every where and Ether at rest compared to Sun.



When change the direction, velocity change

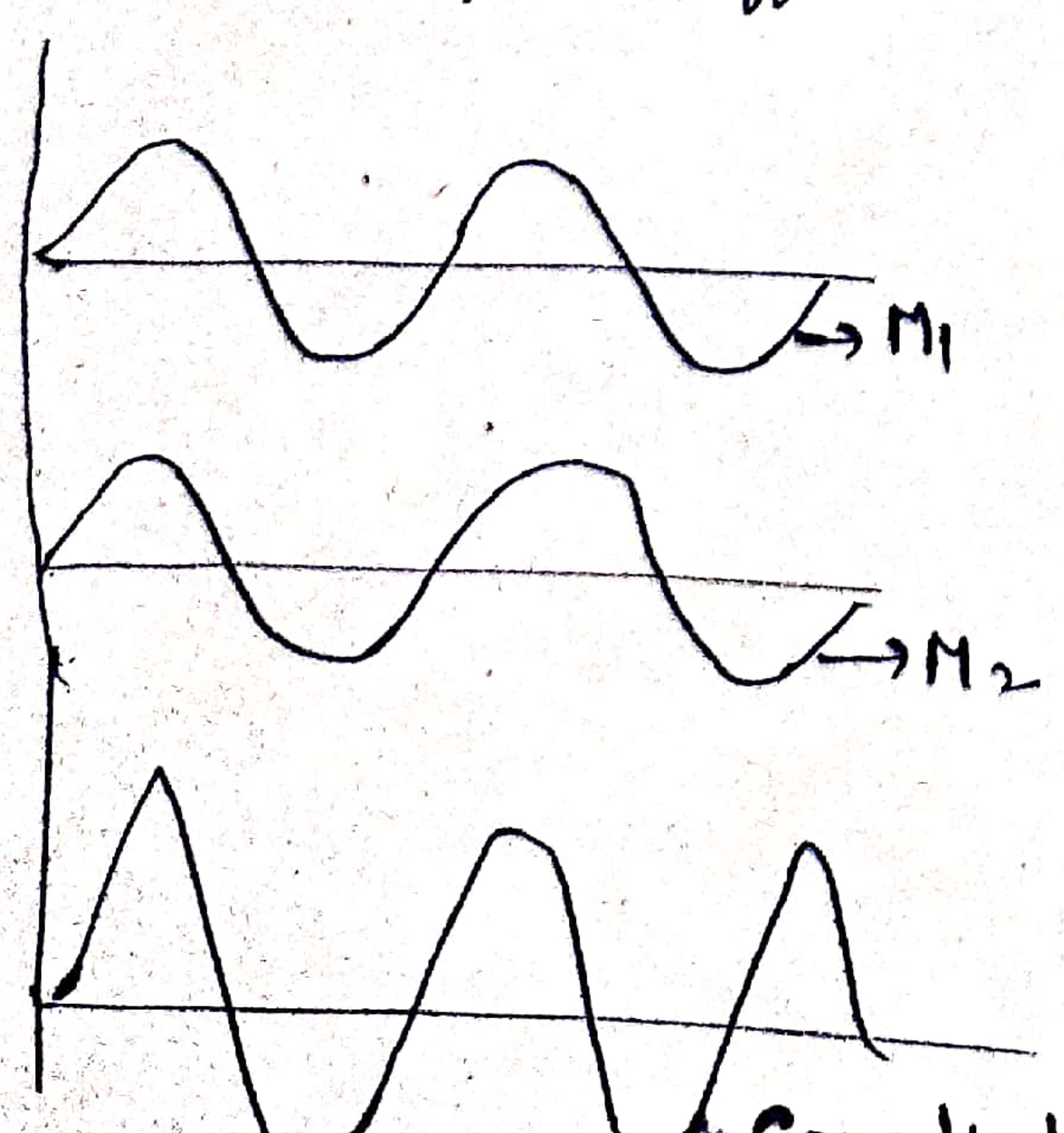
Change in velocity = Create path difference

Path difference due to medium

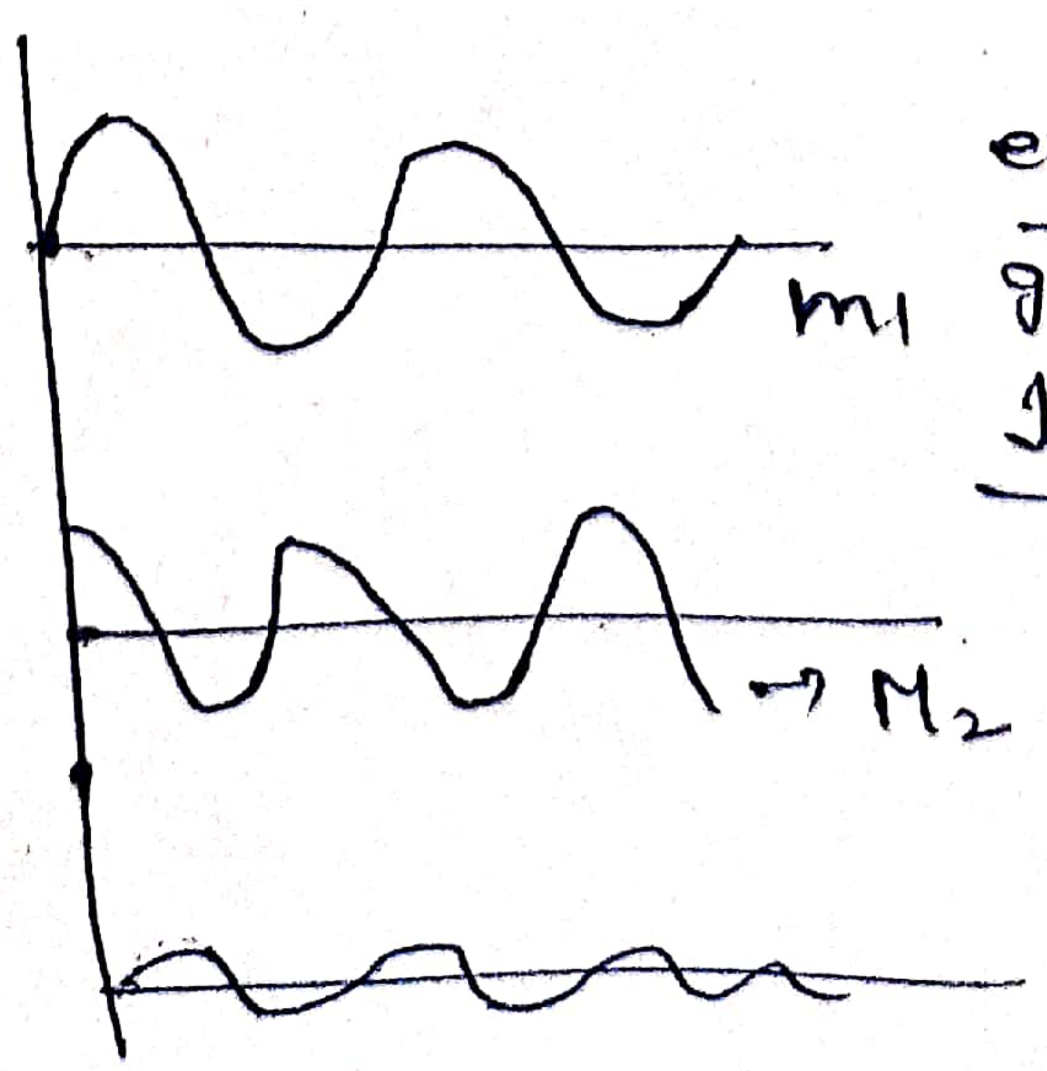


every time get Constructive Interference

After Six month all setup move with 180°.



Constructive Interference



Destructive Interference

Postulates of Special Relativity :

1. The Laws of Physics are the same in all inertial frame of Reference.

2. The Speed of light in free space has the same in all inertial frame of reference

Explaination :→

1. All Laws of Physics are same in even object at rest

OR

Motion with Constant Velocity

Newton's theory of Relativity

2. Speed of light is universal constant for every inertial frame of reference.

Time is absolute ✗

Time is relative. (Speed of light)

Speed of light not depends upon source & observer

for different events, time may be different, [according to classical Time is absolute that means Time is same for all events.

For different frame of reference. Time may be different

Lorentz Transformations :-

* Postulates of Special theory of Relativity
* Space and time homogeneous.

$\vec{r} = ct$

$x^2+y^2+z^2 = c^2t^2$

$x^2+y^2+z^2-c^2t^2=0$ —— (1)

$x'^2+y'^2+z'^2-c^2t'^2=0$ —— (2)

~~Gali~~.

Galilean Transformation

$x' = x - vt$

$y' = y$

$z' = z$

$t' = t$

$x' = x - vt$, $y' = y$, $z' = z$ & $t' = t$ putting in eq^n (2)

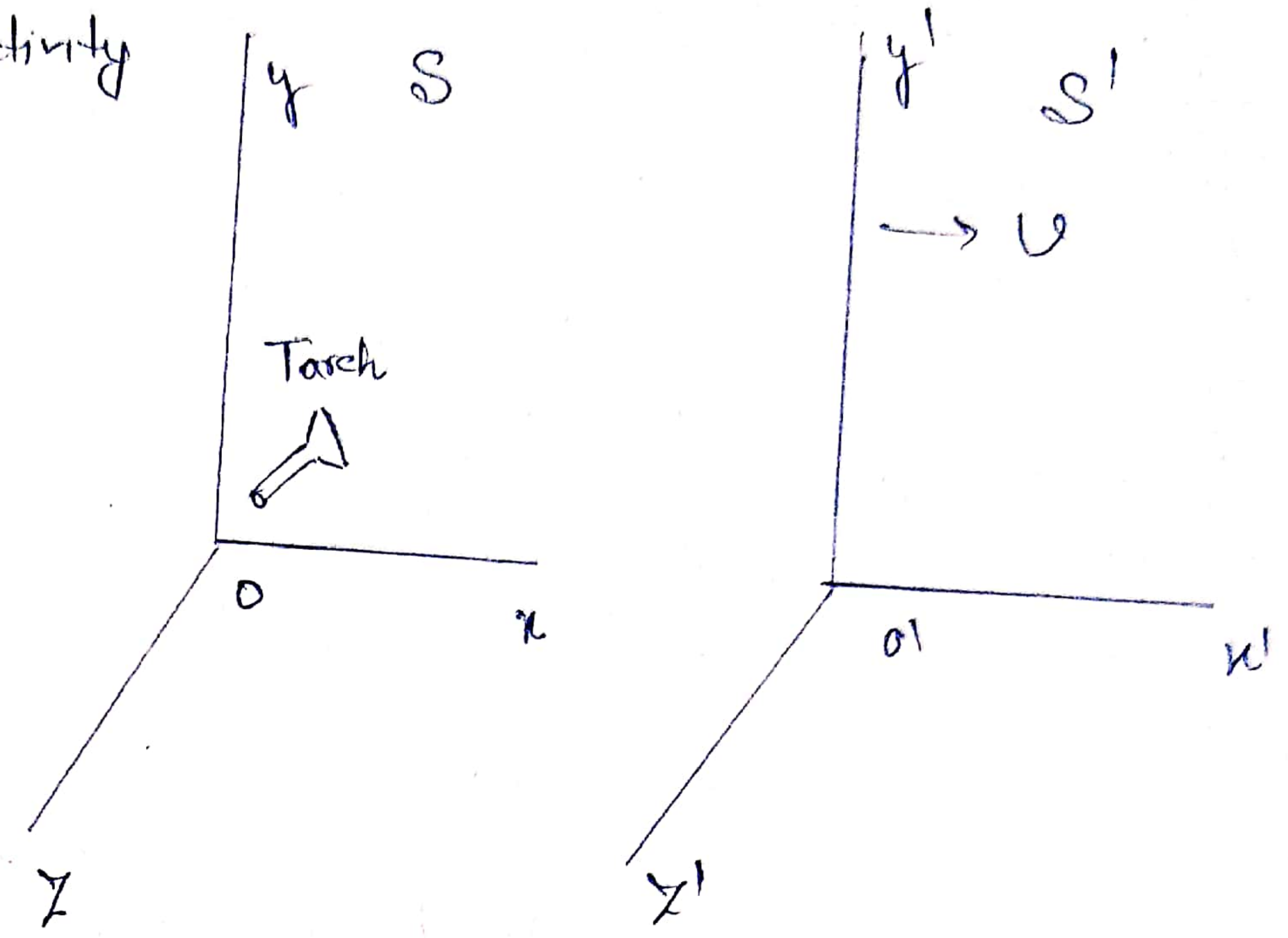
$(x-vt)^2 + y^2 + z^2 - c^2t^2 = 0$

$x^2 + \underbrace{v^2t^2 - 2xvt}_{} + y^2 + z^2 - c^2t^2 = 0$ —— (3)

eq^n (1) & (3) should be same (Not Invariance)

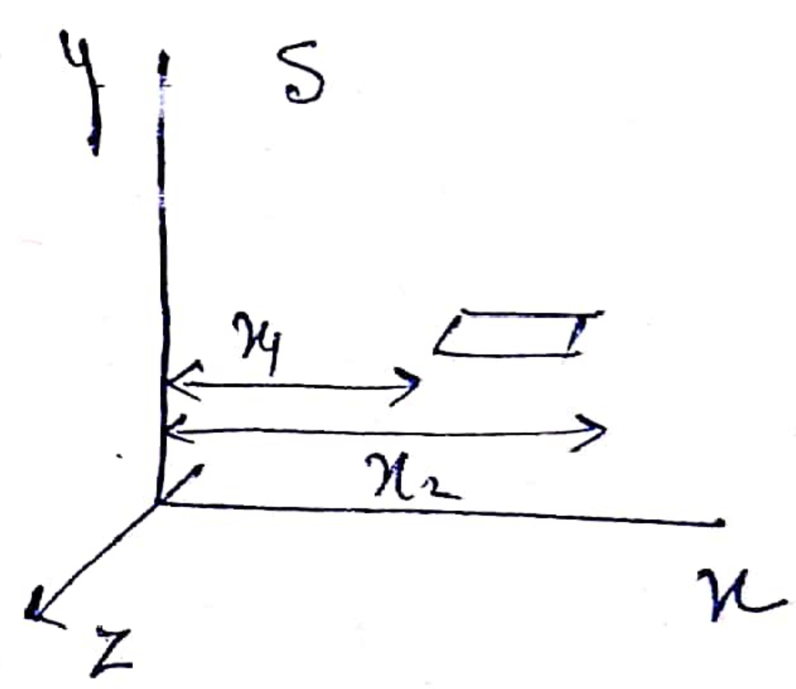
Lorentz have some modification

$$\left.\begin{aligned} x' &= \alpha x + \beta t \\ y' &= y \\ z' &= z \\ t' &= \gamma x + \delta t \end{aligned}\right\} \text{—— (4)}$$



initial time

$\boxed{t = t' = 0}$



$l = x_2 - x_1$

$l' = x_2' - x_1'$

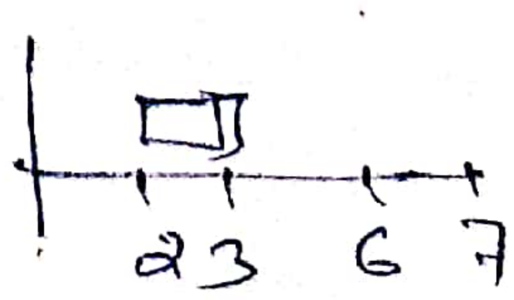
Let $\boxed{x_2 - x_1 = A(x_2' - x_1')}$

Linear Relationship in l & l'

ex. $x_2 - x_1 = A(x_2'^2 - x_1'^2)$

$= A(9-4)$

$= 5A$



$A(7^2-6^2) = 13A$

Same rod ~~but~~ it Voilet the homogenity Condition

Let $x' = 0$

So $\alpha x + \beta t = 0$

$$\frac{dx}{dt} = -\frac{\beta}{\alpha}$$

$$\boxed{v = -\frac{\beta}{\alpha}} \quad (5)$$

$$\frac{dx'}{dt'} = -v \quad (6)$$

$x = 0$

putting in eqn (4)

$$\left.\begin{aligned} x' &= 0 + \beta t \\ t' &= \delta t \end{aligned}\right]$$

$$x' = \frac{\beta}{\delta} \cdot t'$$

$$\frac{dx'}{dt'} = \frac{\beta}{\delta} \quad (7)$$

using eqn. (5) & (6), $\beta = -\alpha v$

$$\frac{dx'}{dt'} = -v$$

$$-v = \frac{-\alpha v}{\delta}$$

$$\boxed{\alpha = \delta} \quad (8)$$

putting the value of β & δ in eqn (4)

$$\left.\begin{aligned} x' &= \alpha x - \alpha v t \\ y' &= y \\ z' &= z \\ t' &= \gamma x + \alpha t \end{aligned}\right] \quad (9)$$

Now putting the values of x', y', z' & t' in eqn (2)

$$[\alpha(x - vt)]^2 + y^2 + z^2 - c^2[\gamma x + \alpha t]^2 = 0$$

$$\alpha^2[x^2 + v^2t^2 - 2xvt] + y^2 + z^2 - c^2(\gamma^2 x^2 + \alpha^2 t^2 + 2\gamma x \alpha t] = 0$$

$$\alpha^2 x^2 + \alpha^2 v^2 t^2 - 2xvt\alpha^2 + y^2 + z^2 - c^2\gamma^2 x^2 - c^2\alpha^2 t^2 - 2\gamma x \alpha t c^2 = 0$$

$$x^2[\alpha^2 - c^2\gamma^2] + xt[-2v\alpha^2 - 2c^2\gamma\alpha] + y^2 + z^2 - t^2[-\alpha^2 v^2 + c^2\alpha^2] = 0 \quad (10)$$

Comparing the Co-efficient of eqn (10) with eqn (1)

$$\boxed{\alpha^2 - c^2\gamma^2 = 1} \; (11.1) \qquad \boxed{2v\alpha^2 - 2c^2\gamma\alpha = 0} \; (11.2)$$

$$\boxed{c^2\alpha^2 - \alpha^2 v^2 = c^2} \; (11.3)$$

from 11.2

$$2v\alpha^2 - 2c^2\alpha\gamma = 0$$

$$\boxed{\gamma = \frac{v\alpha}{c^2}} \quad (13)$$

$$\alpha^2 = \frac{c^2}{c^2 - v^2} = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$\boxed{\alpha = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}} \quad (12)$$

putting the values of α & γ in eqn (9)

$$x' = \frac{(x - vt)}{\sqrt{1 - v^2/c^2}}$$

$$y' = y$$

$$z' = z$$

$$t' = -\frac{\alpha v x}{c^2} + \alpha t = \left[t - \frac{vx}{c^2}\right]\alpha$$

$$t' = \frac{\left[t - \frac{v}{c^2}x\right]}{\sqrt{1 - v^2/c^2}}$$

Lorentz transformation

Verfying the Galileean transfor

Lornetz transformation convert into Galilean transformation

Let assume

$$v <<< c \qquad \frac{v^2}{c^2} \cong 0$$

$$x' = \frac{x - vt}{\sqrt{1 - v^2/c^2}} \qquad 1 - \frac{v^2}{c^2} = 1 - 0$$

$$x' = x - vt$$

$$y' = y$$

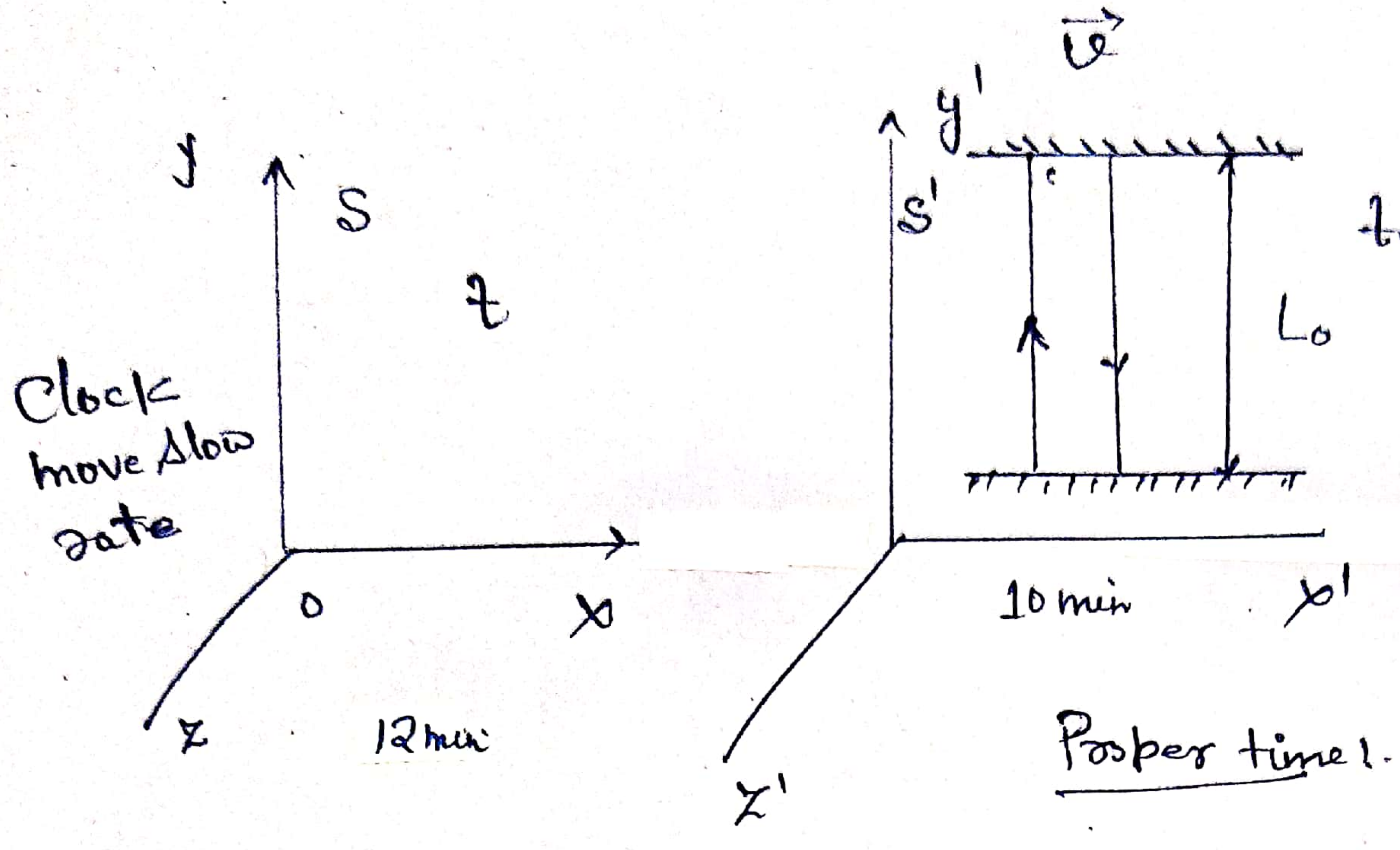
$$z' = z$$

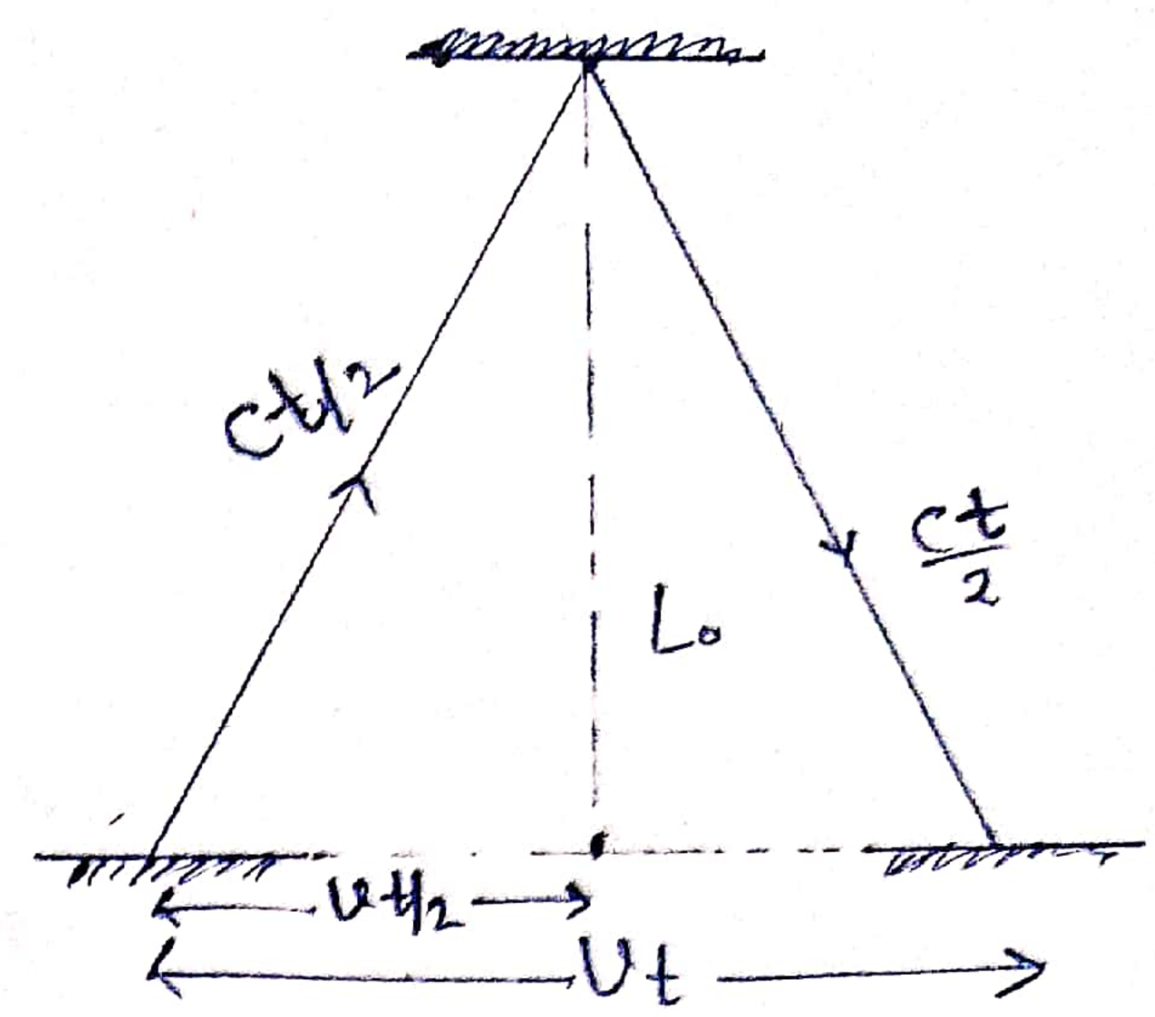
$$t' = \frac{t - \frac{v}{c^2}x}{\sqrt{1 - v^2/c^2}} \qquad \frac{v}{c^2} = 0$$

$$\boxed{t' = t}$$

Time Dilation

Time depends on speed of light





Proper time: For which observer all events happening in same place

- Same event observed by two different observer, which are moving relative to each other, will give you different time interval for the same event.

Temporal and spatial separations are entangled

Difference of time and difference of space are co-related with each other known as time dilation

- For s' frame (proper frame)

$$t_0 = \frac{2L_0}{c} \text{ (proper time)} \quad \text{--- (1)}$$

$$\left(\frac{ct}{2}\right)^2 = \left(\frac{vt}{2}\right)^2 + L_0^2$$

$$\frac{c^2t^2}{4} - \frac{v^2t^2}{4} = L_0^2$$

$$t^2 = \frac{4L_0^2}{c^2 - v^2} = \frac{4L_0^2}{c^2\left[1 - \frac{v^2}{c^2}\right]}$$

$$t = \frac{2L_0}{c\sqrt{1 - v^2/c^2}} \quad \text{--- (2)}$$

from equation (1)

$$\boxed{t = \frac{t_0}{\sqrt{1 - v^2/c^2}}}$$

let $\frac{v}{c} = \beta$ speed parameter

$$\boxed{t = \frac{t_0}{\sqrt{1-\beta^2}}}$$

let $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1-\beta^2}}$

$$\boxed{t = \gamma t_0}$$

$$\boxed{t = \gamma t_0}$$

If we are moving with constant velocity we can not identify we are at rest or in motion.

If clock gives the same time, that means we are identifying the S' is in motion and S is at rest.

$v < c$

$\beta \not< 1$,

$1 - \beta^2 =$

$=$ less than 1

because some is substract from 1

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\text{less than } 1}$$

For

Clock will fast for the observer whose coordinate will not changed.

$\boxed{\gamma > 1}$

If $v \lll c$

$t = t_0$

If $v = c$

$t = \frac{t_0}{0} \Rightarrow$

$\boxed{t = \infty}$

$\boxed{t_0 = 0}$

So we can say

$\boxed{t > t_0}$

Clock will be slow for frame of Reference S

proper time $= 2.200\ \mu s$

Speed of muon $= 0.9994c$

Distance travel by muon

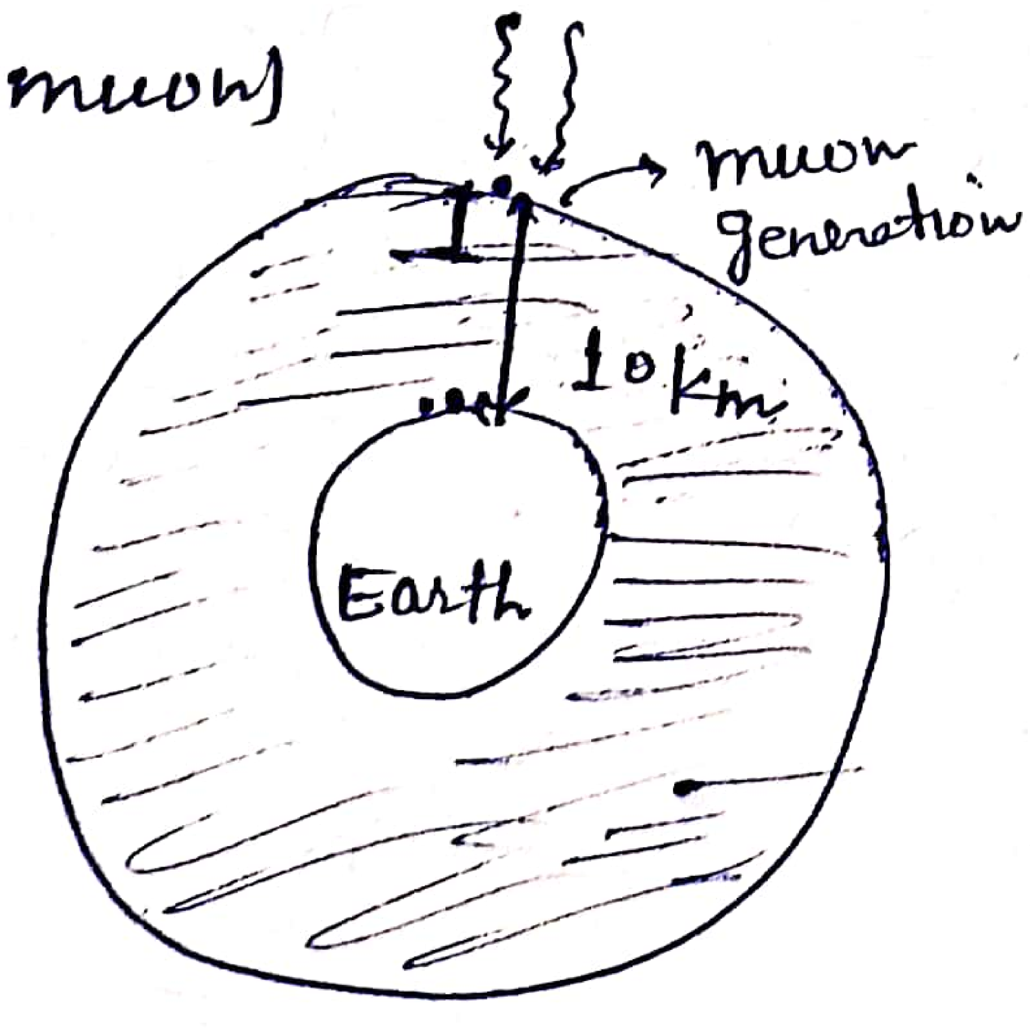
$$x = v \cdot t = 2.200 \times 10^{-6} \times 0.9994 \times 3 \times 10^{8}$$

$\boxed{x = 659\text{ m}}$

Experimental proof of time dilation

μ – meson (muon)

1. Cosmic Rays
2. Laboratory



$$t = \frac{t_0}{\sqrt{1 - v^2/c^2}} = \gamma t_0$$

$\beta = \frac{v}{c} = 0.9994$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = 28.87$$

$t = \gamma t_0 = 28.87 \times 2.200\ \mu s$

$\boxed{t = 63.51\ \mu s}$

$\boxed{x = 19\text{ km}}$

muon life span $= t$

Creation $\rightarrow E_1$

Disintegration $\rightarrow E_2$

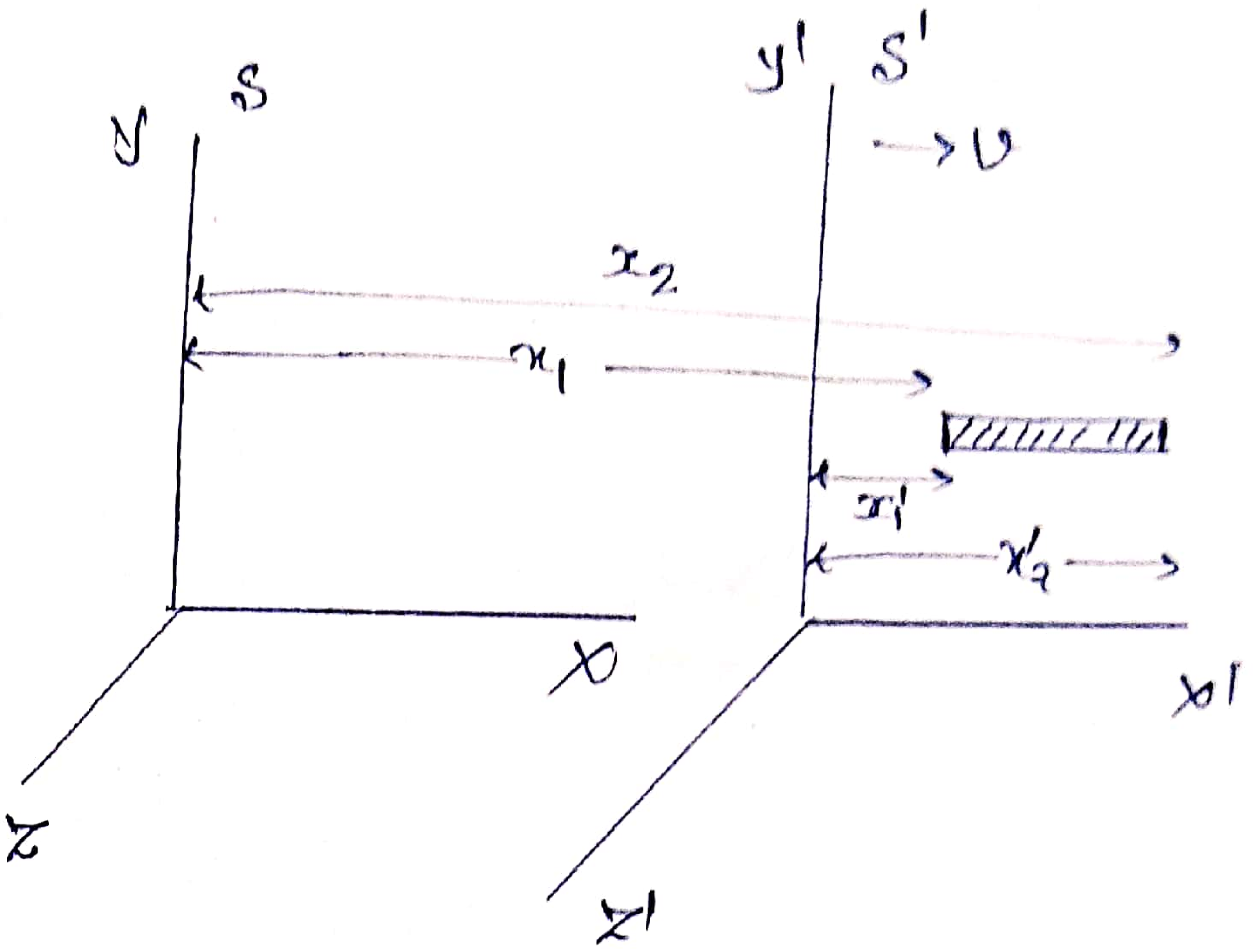
Life span $= 2.200\ \mu s$

$= 2.200 \times 10^{-6}\text{ s}$

Length Contraction

Faster means shorter

Measurement of lengths as well as time intervals are affected by relative motion.



The length L of an object in motion with respect to an observer always appears to the observer to be shorter than its length L_o when it is at rest with respect to him.

$S' \rightarrow$

$$x_2' - x_1' = L_o \text{ (proper length) (Rest length)} \quad (1)$$

frame s

$$x_2 - x_1 = L \text{ (Contracted length)} \quad (2)$$

Simultaneous measurement of x_1 & x_2

Lorentz transformation

$$x_2' = \frac{x_2 - v t_2}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$x_1' = \frac{x_1 - v t_1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (3)$$

From eqn (1) & (3)

$$x_2' - x_1' = \frac{(x_2 - x_1) - v(t_2 - t_1)}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$t_1 = t_2$$

$$L_o = \frac{x_2 - x_1}{\sqrt{1 - v^2/c^2}} = \frac{L}{\sqrt{1 - v^2/c^2}}$$

$$\boxed{L = L_o \cdot \sqrt{1 - v^2/c^2}}$$

$$\sqrt{1 - \frac{v^2}{c^2}} < 1$$

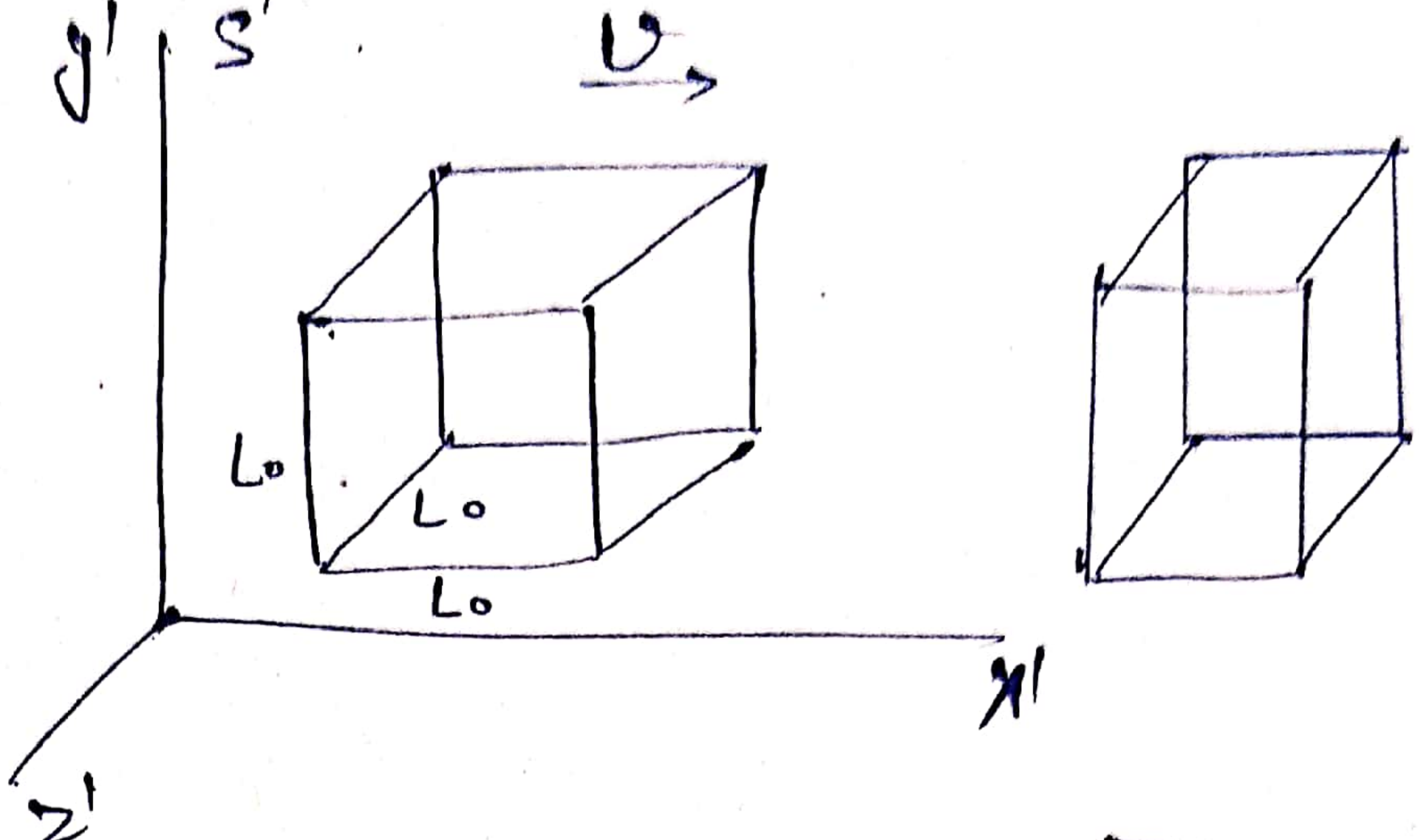
$$\boxed{L < L_o}$$

Length will contract

The contraction is always in the direction of motion

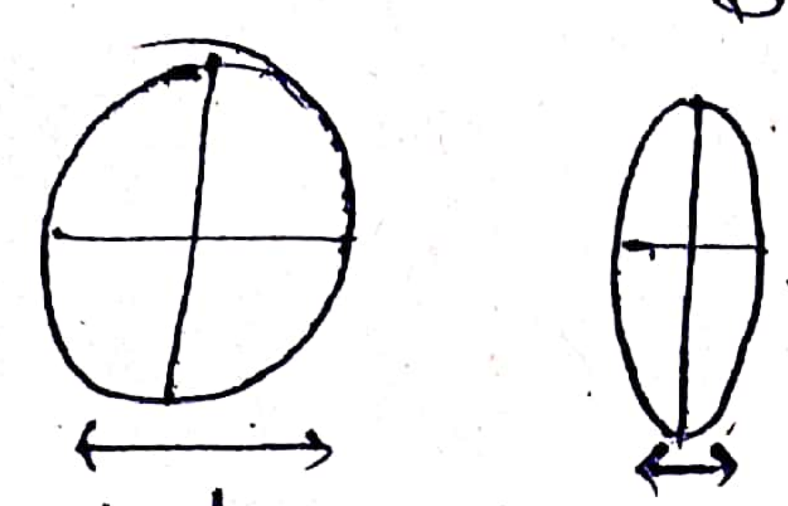
Volume

$V' = L_o^3$



$V' = L_o^3$

$$\boxed{V = L_o^3 \sqrt{1 - \frac{v^2}{c^2}}}$$

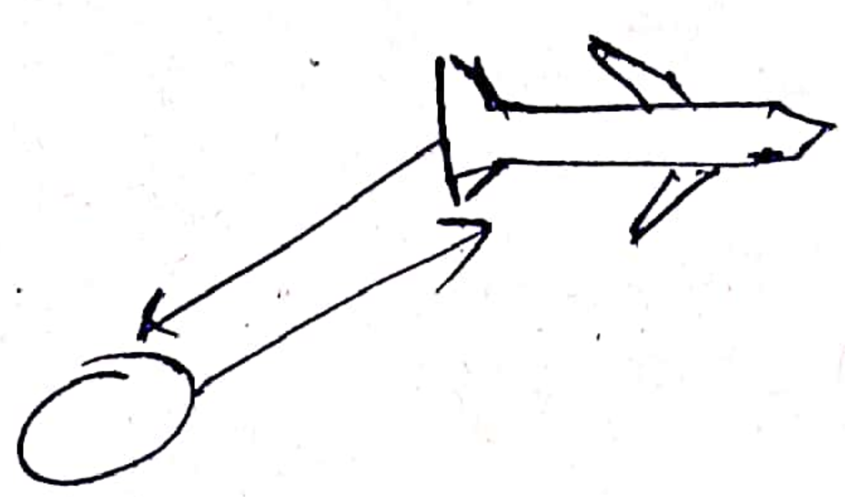


Circle

Reality [illegible] also Relative

SHRINKS

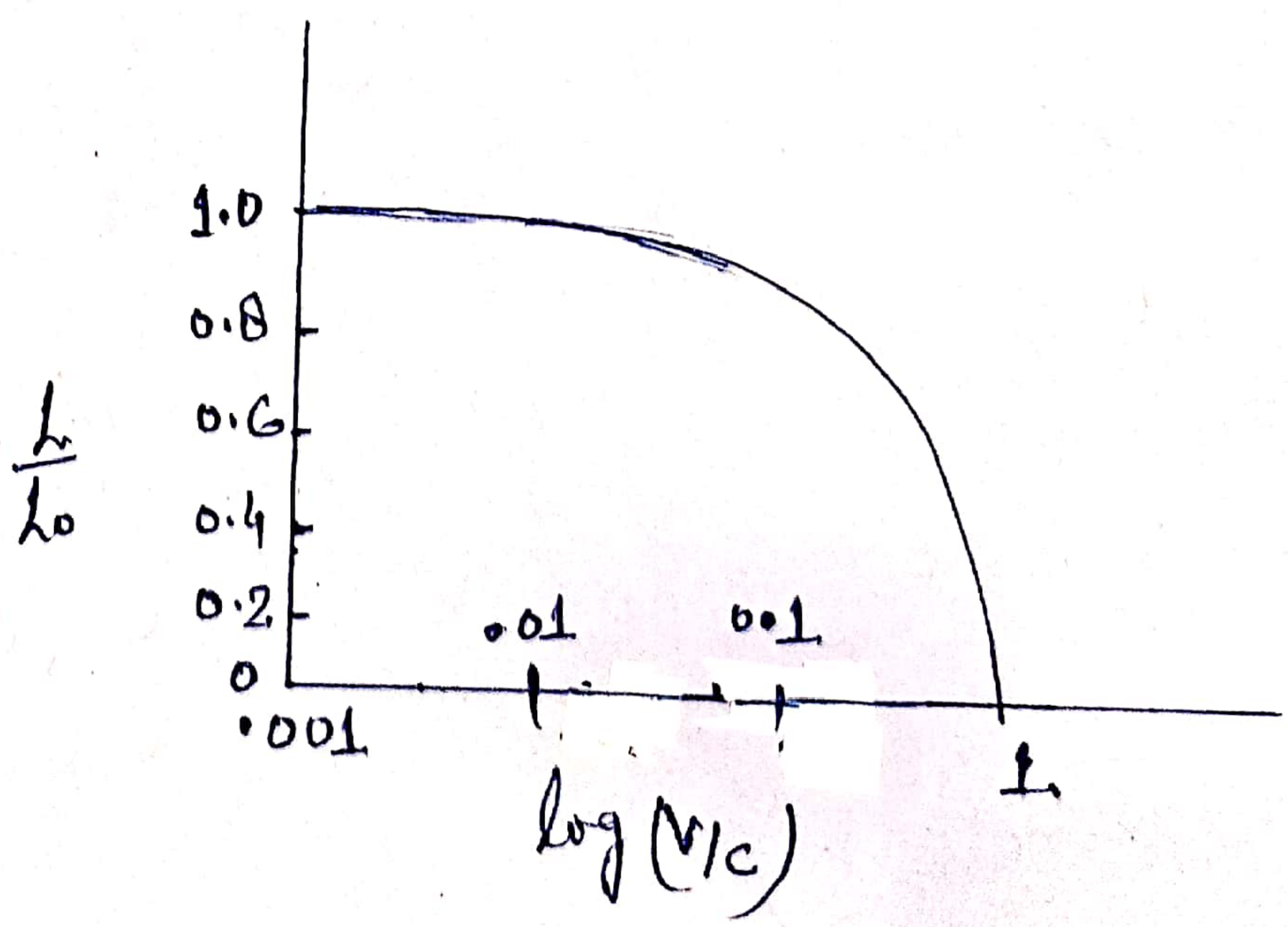
Reciprocal effect



If $v = c$

$L = L_o(0)$

$\boxed{L = 0}$

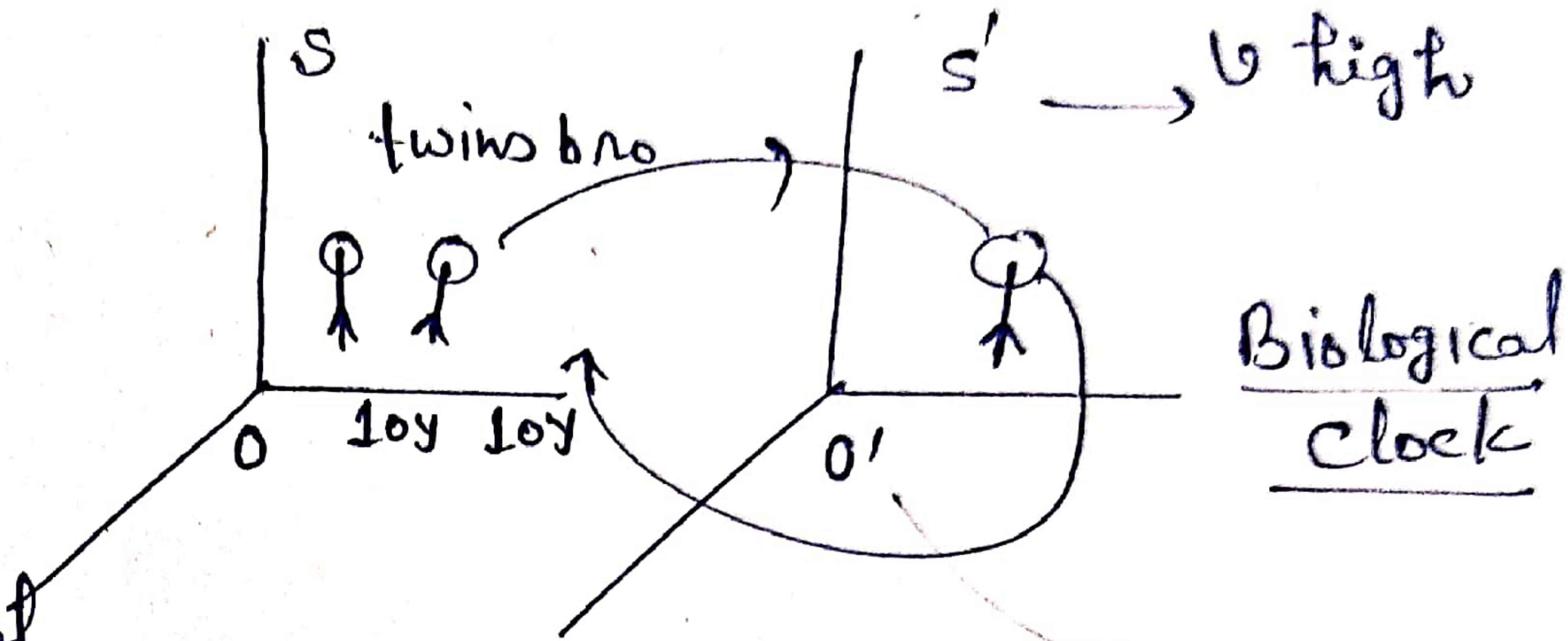


Twins Paradox

The twin paradox is a thought experiment in special relativity involving identical twins, one of whom makes a journey into space in a high speed rocket and returns home to find that the twin who remained on earth has aged more.

Relativistic effect

When the speed of electrons reaches almost that of the speed of light. While obtaining in elliptical path, the electron comes closer to the nucleus. To avoid falling in, it accelerates up to the almost speed of light.



After 10 years

S 10+10 = 20 Y

S' Motion = 12 Y

"A longer life, but it will not seem longer"

- We are now in position to understand the famous relativistic effect known as twin paradox. This paradox involves two identical clocks, one of which remains on the Earth while the other is taken on a journey into space at the speed v and eventually is brought back.

It is usual to replace the clocks with the pair of twins Dick and Jane, a substitution that is perfectly acceptable because the process of life — heart beats, respiration, and so on.

Dick is 20y old, takes a space ~~journey~~ journey at a speed of 0.8c to a star 20 light-years away. To Jane, who stay behind, the pace of Dick life is slower than hers by a factor

$$\sqrt{1-v^2/c^2} = \sqrt{1-(0.80c/c)^2} = 0.60 = 60\%.$$

that means

To Jane, Dick's heart beats only 3 times for every 5 beats of her hearts; Dick takes only 3 breaths for every 5 of hers. Dick thinks only 3 thoughts for every 5 of hers.

Finally Dick returns after 50 years have gone by according to Jane Calender, but to Dick the trip has taken only 30 Y

Dick is thereby only 50 Y old (20+30 Y) whereas Jane, the twin who stayed at home is (20+50 Y) 70 Y old.

Example

Que.:

If A & B are twins, time and both are dies at same year. It is possible, is there any difference in age.?

A B

30 Y 30 Y.

Yes, age of Both are different.

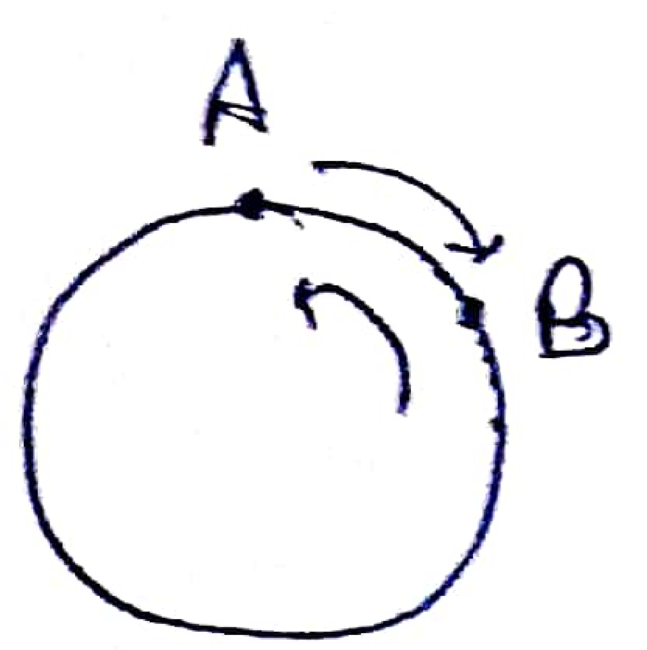
A → 20 Years

B → 20 Years

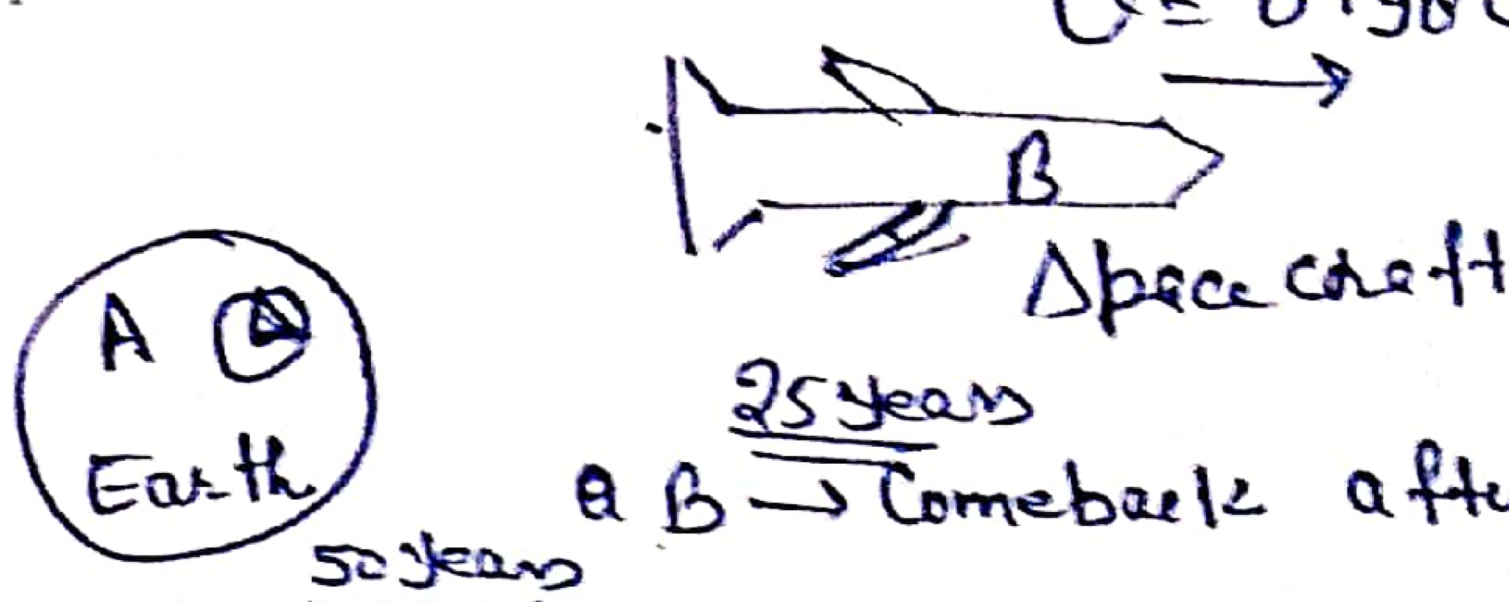
Any object is moving with the speed of light ⇒ time is slow.

That by i.e. B is growing slowly (time slow)

Paradox



$v = 0.98c$



25 Years

a B → Comeback after 30 Year

50 Years

2030

A → Calender Record the Date

Aat Earth ~~watch~~ clock

A → 20+30 = 50 / years

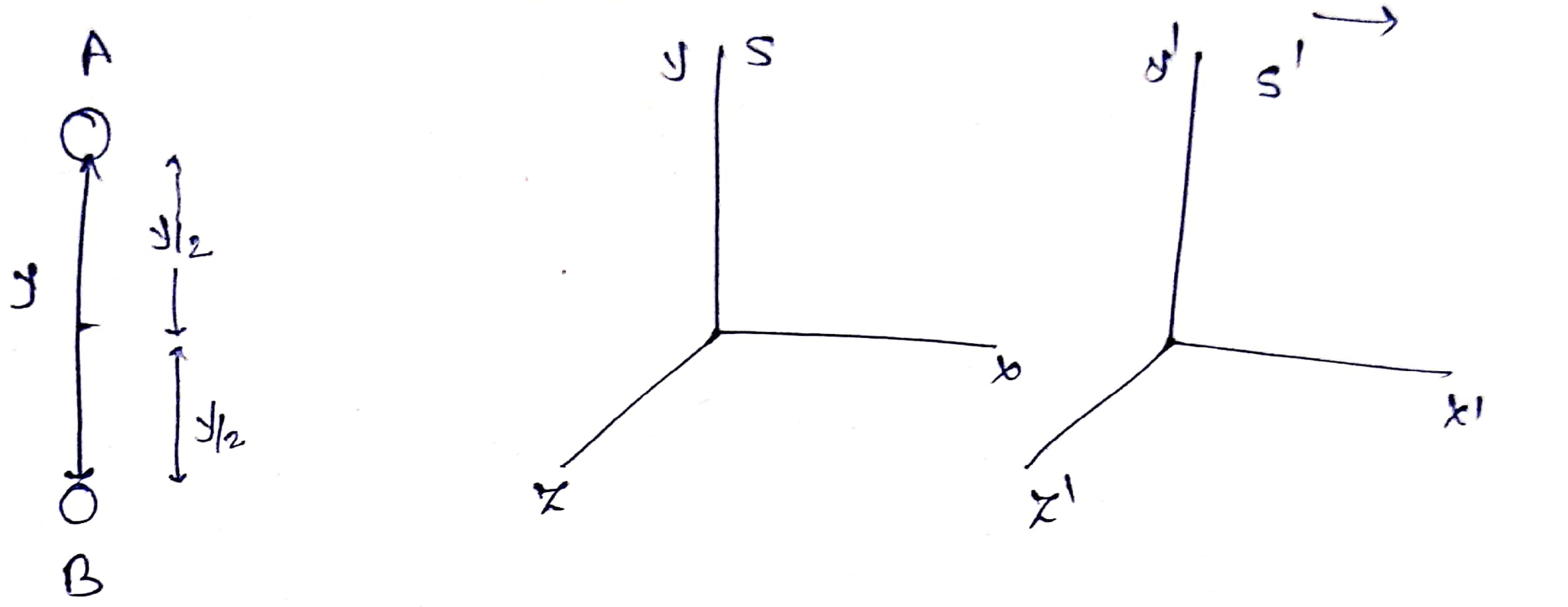
B → 25 Years.

motion is relative means distance is increase.

why the effect only B

If any spacecraft is moving with the speed of light (comparable of speed of light) (0.98c) time is slow at atomic level or molecule level.

Relativity of mass

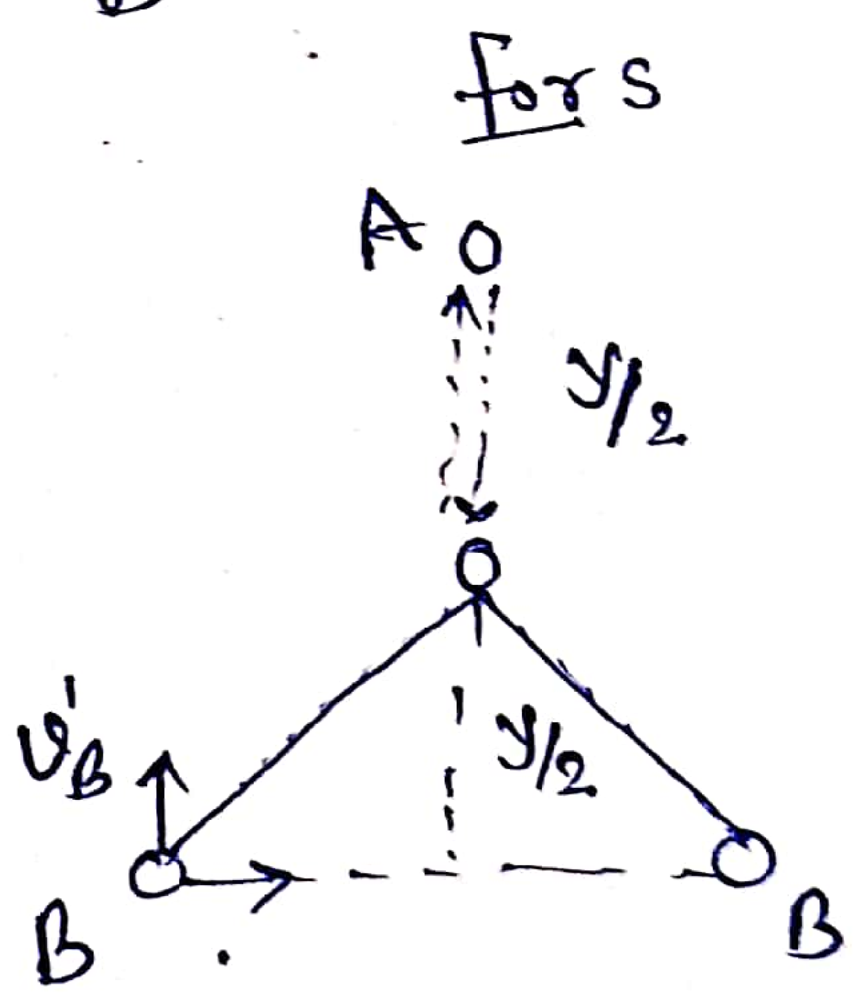


- elastic Collision [Linear momentum and energy are conserved]

A & B are two identical particles (mass, shape are same)

$V_A = V_B'$ (There is no gravity)

The Collision takes place at $y/2$

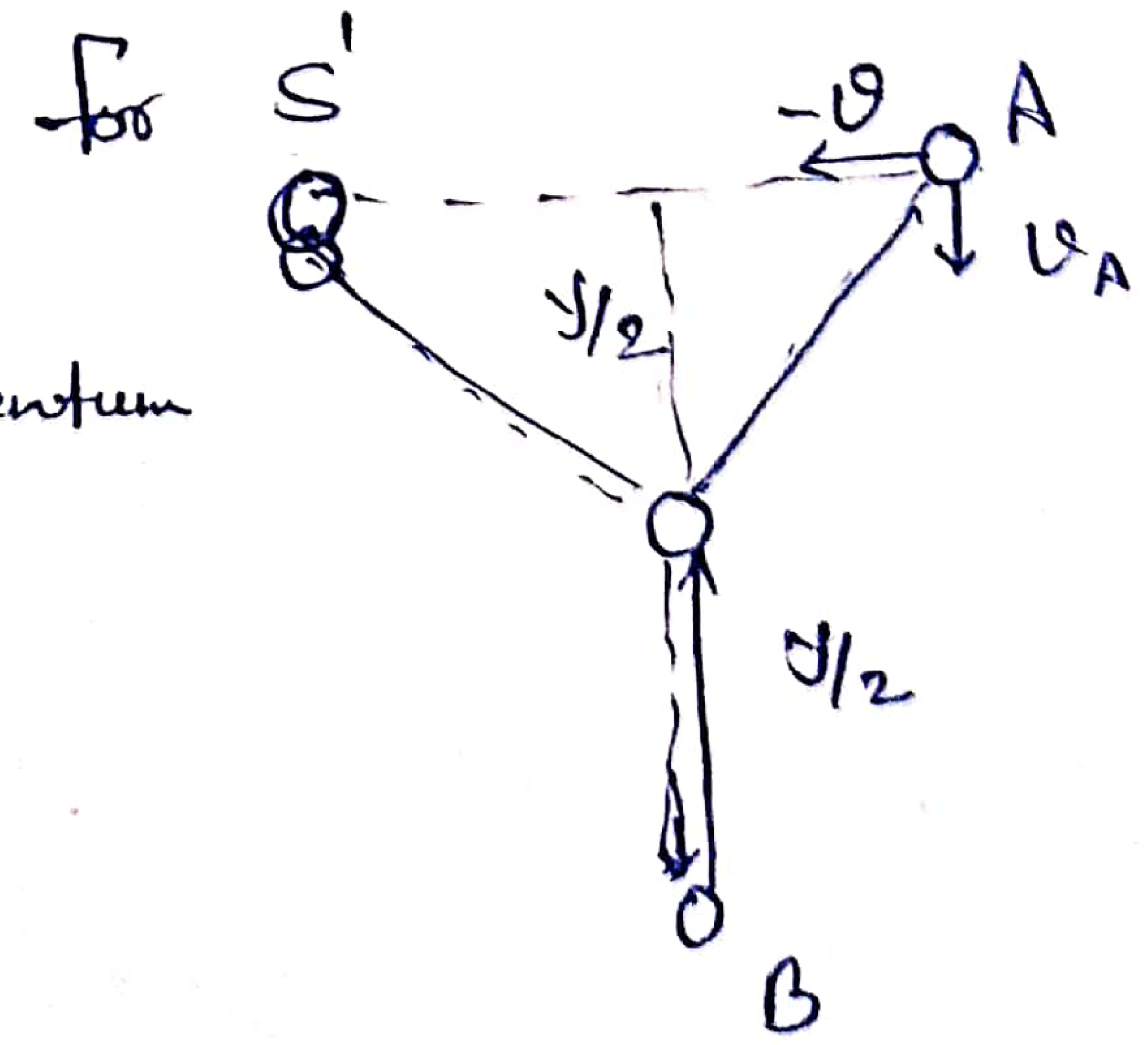


Time taken by 'A' in round trip is for S

- $T_0 = \dfrac{y}{V_A}$

For S' Time taken by 'B'

$T_0 = \dfrac{y}{V_B'}$



Applying conservation of Linear momentum in S frame

$m_A v_A = m_B V_B$

$V_B = \dfrac{y}{T}$

$T = T_0 / \sqrt{1 - v^2/c^2}$

$$V_B = \frac{y}{T_0}\sqrt{1-v^2/c^2}$$

$$V_A = \frac{y}{T_0}$$

$$m_A V_A = m_B V_B$$

$$m_A \cdot \frac{y}{T_0} = m_B \frac{y}{T_0}\sqrt{1-v^2/c^2}$$

$$m_A = m_B\sqrt{1-\frac{v^2}{c^2}} \text{ for 'S'}$$

$m_A = m_0$ (Rest)

$m_B = m$ (motion)

$$m_0 = m\sqrt{1-\frac{v^2}{c^2}}$$

$$\boxed{m = \frac{m_0}{\sqrt{1-v^2/c^2}}}$$

$$\boxed{m > m_0}$$

~~for s'~~

1908

- e/m ratio for fast moving particles are smaller than slow moving electrons.

If $v \to c$

$$\frac{v^2}{c^2} \approx 1$$

$$\boxed{m \to \infty}$$

It is not possible

No particle have the infinite mass

No one particle have the speed equal to the speed of light

for massy particle

$$\boxed{v < c}$$

Relativistic momentum

$$p = mv$$

$$p = \frac{m_0 v}{\sqrt{1-v^2/c^2}}$$

$$\boxed{p = \gamma m_0 v}$$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

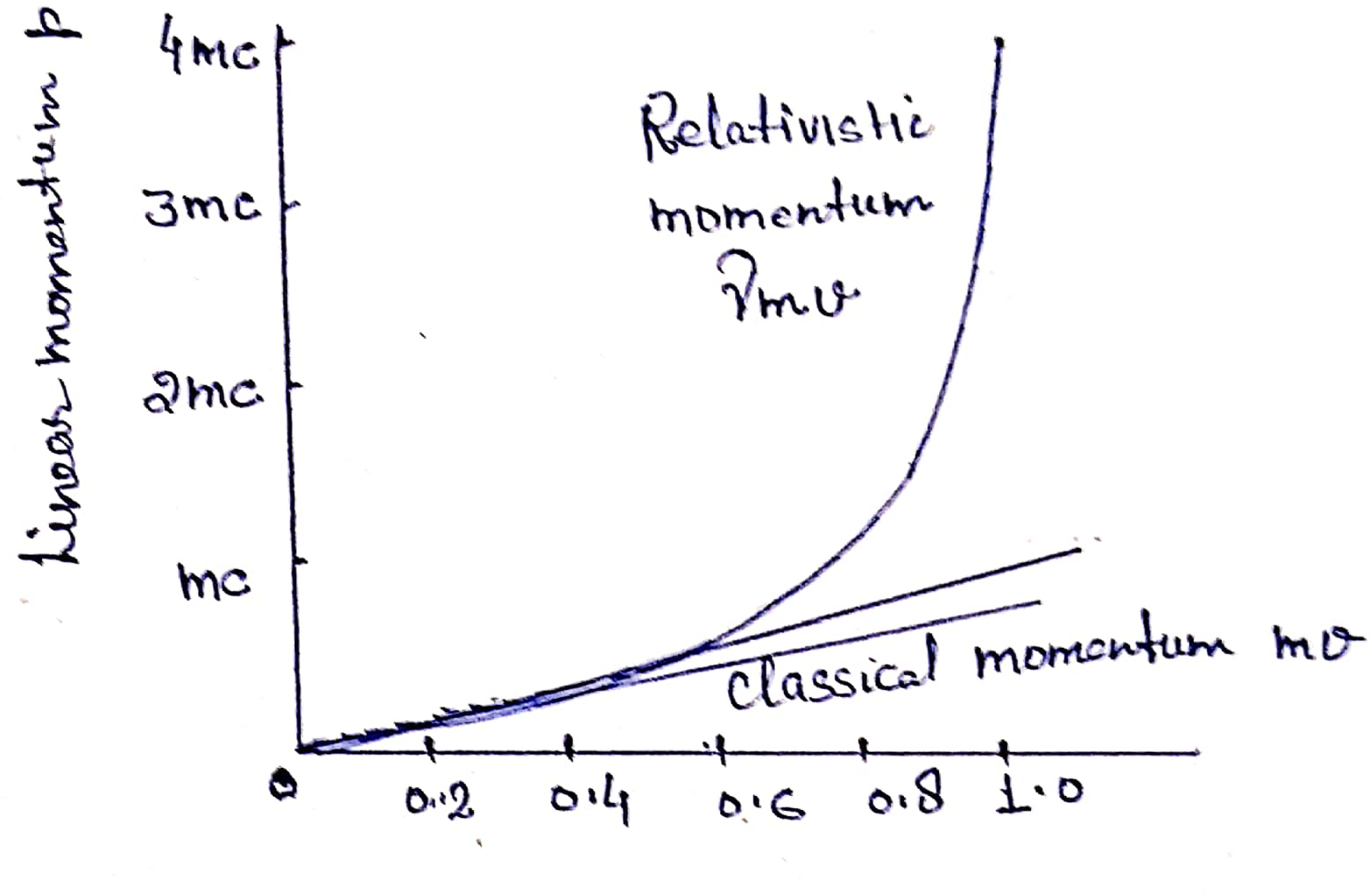
$m_0 \to$ Proper mass (or Rest mass)

Relativistic Second law

In relativity Newton's second law of motion is given by

$$F = \frac{dP}{dt} = \frac{d}{dt}(\gamma m v)$$

This is more complicated than the classical formula $F = ma$ because γ is a function of v. When $v \ll c$, γ is very nearly equal to 1, and F is very nearly equal to mv

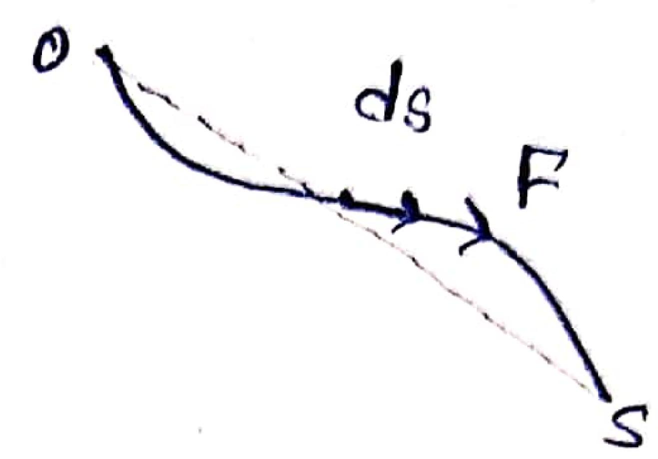


The momentum of an object moving at the velocity v relative to an observer. The mass m of the objects is its value when it is at rest relative to obserber. The object velocity can never reach c because its momentum would then be infinite, which is impossible.

The relativistic momentum $\gamma m v$ is always correct; the classical momentum mv is valid for velocities much smaller than c.

Mass and Energy Equivalance

$$K.E = \int_0^S F\, ds$$



$$F = \frac{d}{dt}(mv)$$

$$K.E = \int_0^S \frac{d}{dt}(mv)\, ds$$

$$K.E = \int_0^{mv} v\, d(mv)$$

$$m = \frac{m_0}{\sqrt{1-v^2/c^2}}$$

$$K.E = \int_0^v v\, d\left(\frac{m_0 v}{\sqrt{1-v^2/c^2}}\right)$$

$$\int x\,dy = xy - \int y\,dx$$

$$K.E = \frac{v\, m_0 v}{\sqrt{1-v^2/c^2}} - \int_0^v \frac{m_0 v}{\sqrt{1-v^2/c^2}}\, dv$$

$$K.E. = \frac{m_0 v^2}{\sqrt{1-v^2/c^2}} - m_0 \int_0^v \left(1-\frac{v^2}{c^2}\right)^{-1/2} v\, dv$$

$$\int f^m(x) \cdot f'(x)\, dx = \frac{f^{(n+1)}(x)}{(n+1)}$$

$$K.E = \frac{m_0 v^2}{\sqrt{1-v^2/c^2}} - m_0 \int_0^v \left(1-\frac{v^2}{c^2}\right)^{-1/2} \left(\frac{-2v}{c^2}\right) \cdot \left(-\frac{c^2}{2}\right) dv$$

$$K.E. = \frac{m_0 v^2}{\sqrt{1-v^2/c^2}} - m_0\left[\frac{(1-v^2/c^2)^{-1/2+1}}{-1/2+1}\left(-c^2/2\right)\right]_0^v$$

$$K.E. = \frac{m_0 v^2}{\sqrt{1-v^2/c^2}} + m_0\left[c^2\sqrt{(1-v^2/c^2)}\right]_0^v$$

$$K.E = \frac{m_0 v^2}{\sqrt{1-v^2/c^2}} + m_0\left[c^2\sqrt{1-v^2/c^2} - c^2\right]$$

$$= \frac{m_0 v^2}{\sqrt{1-v^2/c^2}} + m_0 c^2\sqrt{1-v^2/c^2} - m_0 c^2$$

$$= \frac{m_0 v^2 + m_0 c^2\left[1-\frac{v^2}{c^2}\right] - m_0 c^2\sqrt{1-v^2/c^2}}{\sqrt{1-v^2/c^2}}$$

$$K.E. = \frac{m_0 c^2}{\sqrt{1-v^2/c^2}} - m_0 c^2$$

$K.E = \gamma m_0 c^2 - m_0 c^2$

$= (\gamma - 1) m_0 c^2$

$$\boxed{K E = mc^2 - m_0 c^2}$$

Total Energy

$= mc^2$

$= \frac{m_0 c^2}{\sqrt{1-v^2/c^2}}$

$$\boxed{T.E = \gamma m_0 c^2}$$

$$\boxed{mc^2 = K.E. + m_0 c^2}$$

$E_0 = m_0 c^2 \rightarrow$ Rest Energy

$mc^2 \rightarrow$ Total energy

$E = K.E. + m_0 c^2$

$$\boxed{E = mc^2}$$

Mass energy equivalance Relation

$E = mc^2$

Energy form and Matter form

Mass energy

Energy Can be created or can be destroyed

When energy is created $\Rightarrow$ Some loss of mass

" " lost $\Rightarrow$ must be creation of mass

ex - Pair Production

Mass and energy are the two different form of the same entity example: A stationary body explodes into two fragments each of mass 1.0 kg that move apart at speeds of 0.6c relative to the original body. Find the mass of the original body.

Solution:-

The rest energy of the original body must equal the sum of the total energies of the fragments.,

$$= \gamma m_1 c^2 + \gamma m_2 c^2$$

$$E_o = mc^2 = \frac{m_1 c^2}{\sqrt{1-v^2/c^2}} + \frac{m_2 c^2}{\sqrt{1-v^2/c^2}}$$

$$mc^2 = \frac{2 \times m_1 c^2}{\sqrt{1-v^2/c^2}}$$

$$m = \frac{2 \times 1\,kg}{\sqrt{1-(.6)^2}} = \frac{2 \times 1}{0.8} = 2.5\ kg$$

Important Point:-

The conversion factor between the unit of mass (the Kilogram, kg) and the unit of energy (the Joule J) is c^2

So, 1 kg of matter - has energy content of mc^2 ⇒

$$mc^2 = 1\,kg \times (3\times10^8\, m/s)^2$$

$$= 1 \times 9\times10^{16}\ \text{Joule}$$

$$= 9\times10^{16}\ \text{Joule}$$

This is enough energy to send a payload of a million tons to the moon.

In every chemical reaction that envolves energy, a certain amount of matter disappears, but the lost mass is so small a fraction of the total mass of the reacting substances that it is imperceptible only about 6×10^{-11} kg of matter vanishes when 1 kg of dynamite explods, which is impossible to measure directly but the more than 5 millions Joule of energy that is released is hard to avoid noticing.

Kinetic Energy at low Speeds

When the relative speed v is small compared with 'c', the formula for K.E. must reduce to the familiar $\frac{1}{2}m_0v^2$,

$$K.E. = mc^2 - m_0c^2 = \gamma m_0c^2 - m_0c^2 \qquad m = \gamma m_0$$

$$= \frac{1}{\sqrt{1-v^2/c^2}} m_0c^2 - m_0c^2$$

$$= m_0c^2 (1 - v^2/c^2)^{-1/2} - m_0c^2$$

$$= m_0c^2 (1 + \tfrac{1}{2} v^2/c^2) - m_0c^2$$

$$= m_0c^2 + \tfrac{1}{2}m_0v^2 - m_0c^2$$

$$\boxed{K.E. = \tfrac{1}{2}m_0v^2} \qquad \boxed{v \ll c}$$

Energy and momentum :

"How they fit togeather in relativity"

Total energy and momentum are conserved in an isolated system and the rest energy of a particle is invariant.

Total energy $E = mc^2 = \gamma m_0c^2$

$$E = \frac{1}{\sqrt{1-v^2/c^2}} m_0c^2$$

$$E^2 = \frac{m_0^2c^4}{(1-v^2/c^2)} \quad \text{--- (1)}$$

momentum $p = \gamma m_0 v$

$$p = \frac{m_0v}{\sqrt{1-v^2/c^2}}$$

$$p^2c^2 = \frac{m_0^2v^2c^2}{(1-v^2/c^2)} \quad \text{--- (2)}$$

Eqn (1) - eqn (2)

$$E^2 - p^2c^2 = \frac{m_0^2c^4 - m_0^2v^2c^2}{1-v^2/c^2}$$

$$E^2 - p^2c^2 = \frac{m_0^2c^4[1 - v^2/c^2]}{1-v^2/c^2}$$

$$E^2 - p^2c^2 = (m_0c^2)^2$$

$$\boxed{E^2 = (m_0c^2)^2 + (pc)^2}$$

We note that, because mc^2 is invariant so $E^2 - p^2c^2$; this quantity for a particle has the same value in all frame of reference.

Mass less particles

Zero Rest Mass particles

In classical mechanisms, a particle must have rest mass in order to have energy and momentum but in relativistic mechanics, this requirement does not hold.

The relativistic formulas for total energy (E) and linear momentum is p are given by

$$E = mc^2 = \gamma m_o c^2 = \frac{m_o c^2}{\sqrt{1 - v^2/c^2}}$$

$$p = mv = \frac{m_o v}{\sqrt{1 - v^2/c^2}}$$

$$E = pc$$

$$mc^2 = mvc$$

$$\boxed{v = c}$$

Thus a particle of zero rest mass always moves with the speed of light.

$$m = \frac{m_o}{\sqrt{1 - v^2/c^2}}$$

$$v = c$$

$$m = \frac{m_o}{\sqrt{1-1}} = \frac{m_o}{0}$$

$$\boxed{m_o = 0}$$

when $m_o = 0$ and $v < c$, it is clear that

$$E = 0, \quad p = 0.$$

Thus a mass less particle with a speed less than speed of light can have neither energy nor momentum. However, when $m_o = 0$ and $v = c$,

$$E = \frac{0\, c^2}{\sqrt{1-1}} = \frac{0}{0} \quad \text{OR} \quad p = \frac{0}{0}$$

which are indeterminate i.e. E and p can have any value. Thus mass less particles possess energy and momentum provided they travel with the speed of light.

According energy and momentum relation

$$E = \sqrt{p^2 c^2 + m_o^2 c^4},$$

But for a particle of zero rest mass like photon and neutrino $m_o = 0$

example 1

An electron ($m = 0.511\ MeV/c^2$) and a photon ($m=0$) both have momenta of $2.000\ MeV/c$ find the total energy of each.

Solⁿ

Electron total energy

$$E = \sqrt{m^2c^4 + p^2c^2} = \sqrt{\quad}$$

$$= \sqrt{[0.511\ MeV/c^2]^2 c^4 + (2\ MeV/c)^2 c^2}$$

$$= \sqrt{(0.511)^2 + 4}\ MeV$$

$$\boxed{E = 2.064\ MeV}$$

Photon total energy

$$E = \sqrt{p^2c^2 + m_0^2c^4}$$

$$m_0 = 0$$

$$E = pc = 2.00\ MeV/c \times c$$

$$\boxed{E = 2.0\ MeV}$$

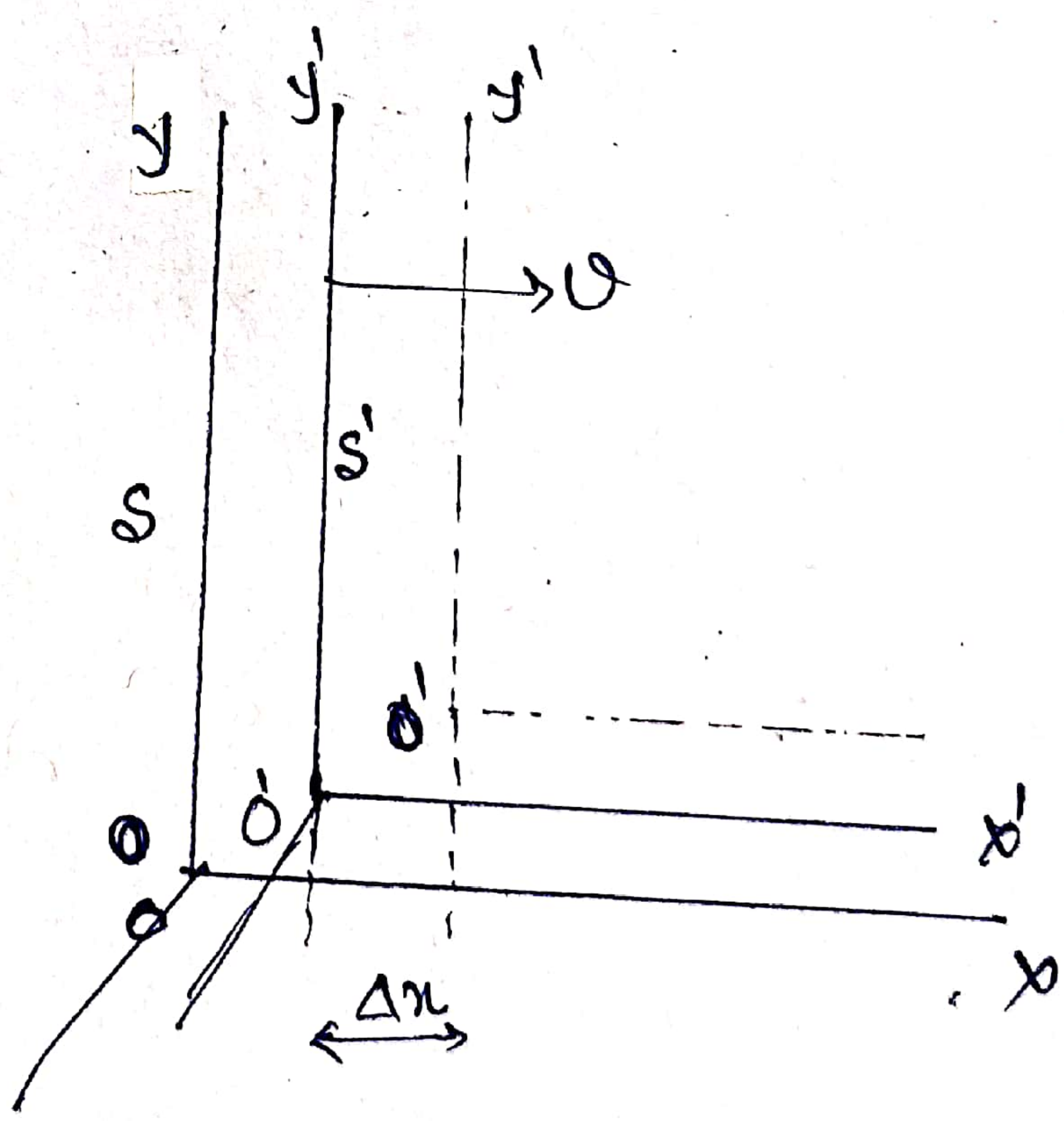
Relativistic Doppler Effect :→

We are all familier with the increase of in pitch of sound when its source approaches us (or we approach the source) and the decrease in pitch when the source recedes from us (or we recede from the source). These change in frequency constitute the doppler effect.

Whenever there is a relative motion between a source of light and observer, the apparent frequency of light observed is different from the actual frequency of light emitted by the source. The phenomenon of apparent change in frequency of light due to relative motion between the source of light and the observer is called doppler effect.

In relativity, Doppler effect may be longitudinal or and transverse.

Longitudinal Doppler effect :→

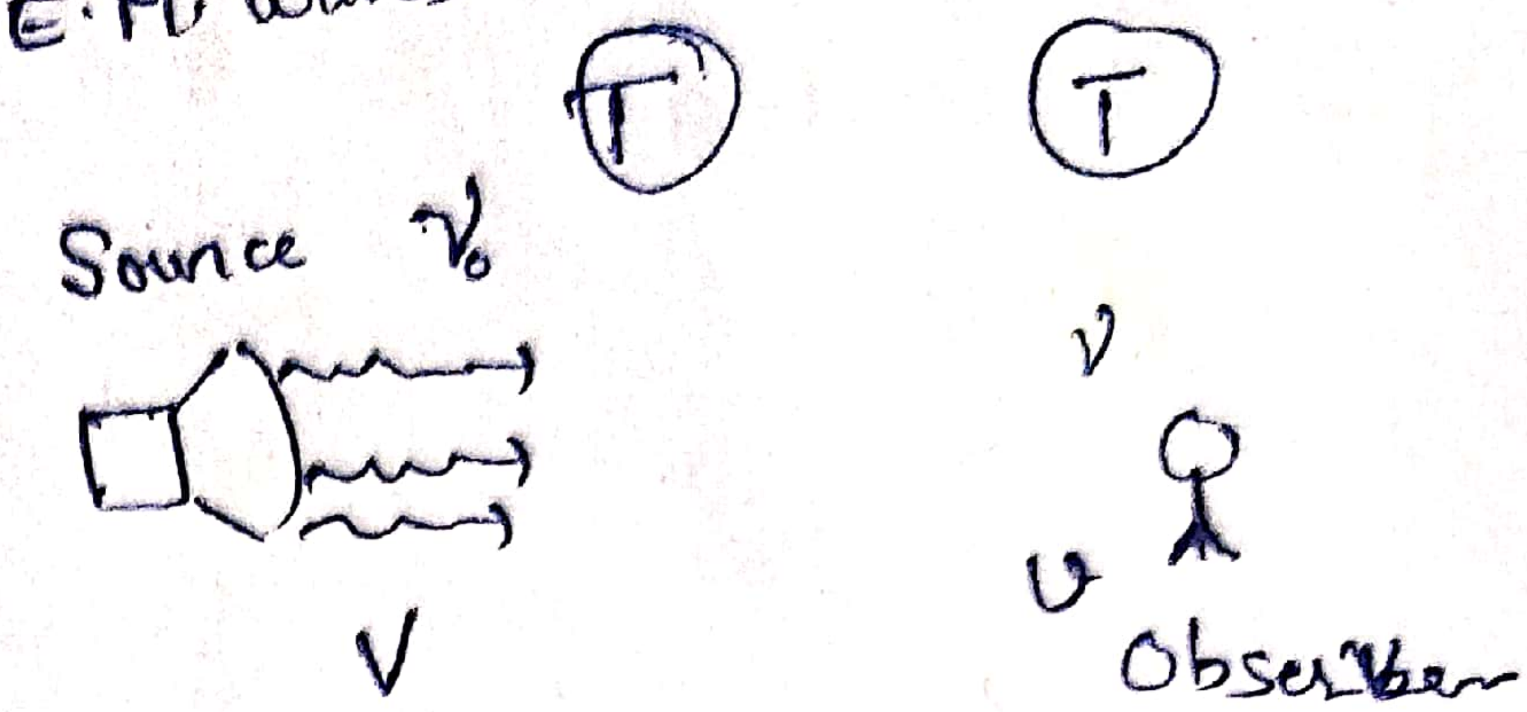


Consider two inertial frame of reference S and S', such that S' is moving with the respect to S with uniform velocity v along positive direction of x-axis.

Let two pulses or light signals be emitted from a source placed at the origin O in frame S at time $t=0$ and $t=T$ where T is true (proper) time period of the light pulses. Let $\Delta t'$ be the interval between the reception of these pulses by an observer at the origin O' in frame S'. Thus apparent time period of the pulses for the observer in frame S' will be

$$\Delta t' = T'$$

Relativistic Doppler effect [E.M. waves]



$\nu_0 \rightarrow$ actual frequency

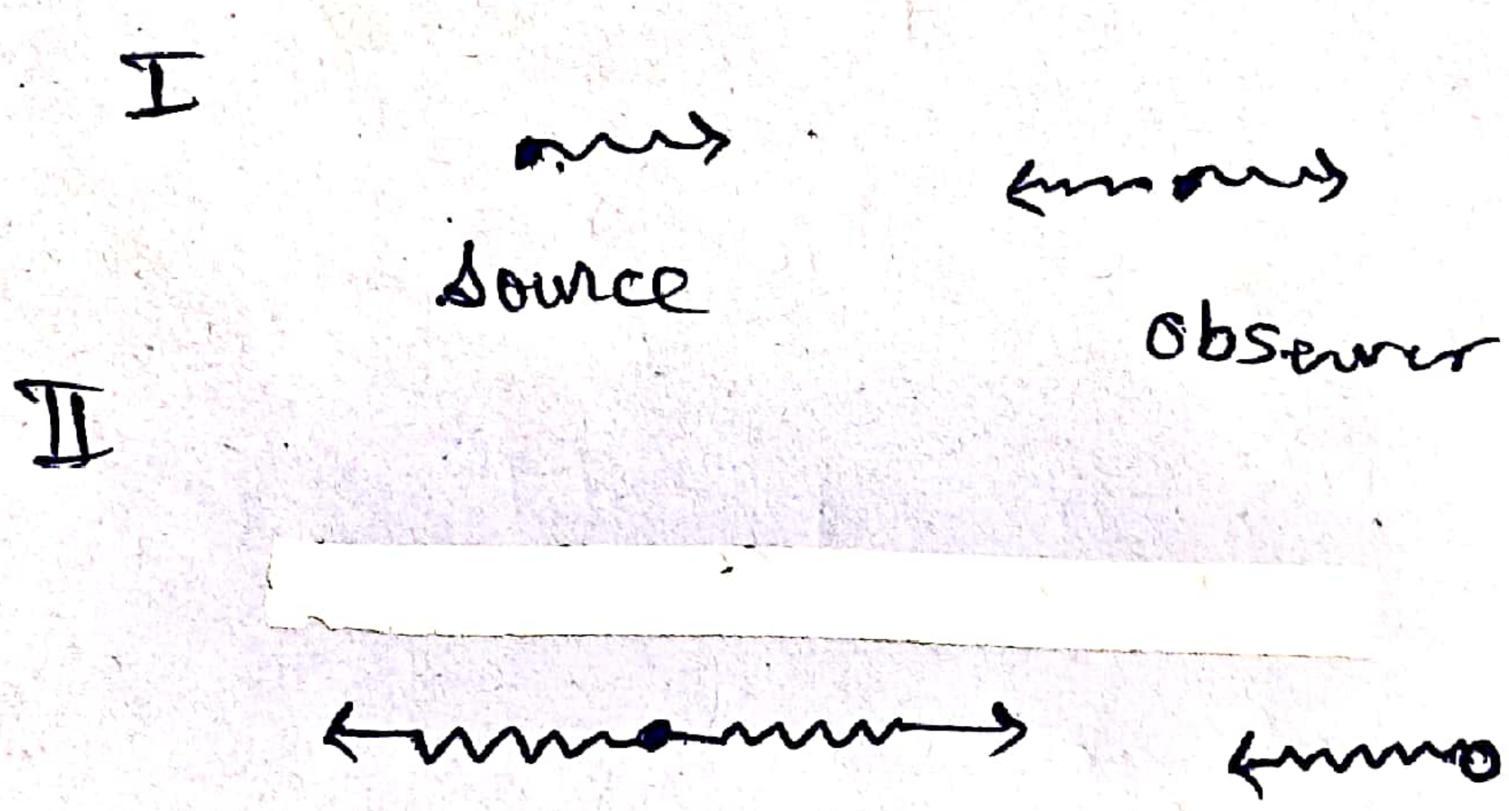
$\nu \rightarrow$ observed frequency

$$\nu = \nu_0 \left(\frac{1 + v/c}{1 - V/c}\right)$$ Classical doppler effect

$v \rightarrow$ speed of observer '+ for motion toward the source'
'− for " away from it'

$V \rightarrow$ speed of source '+ for motion toward the observer'
'− for motion away from him'

Source and observer are absolute motion



Actually we assume
Time is same (S 4 0)

Time period →
First Crest and Second Crest (T)

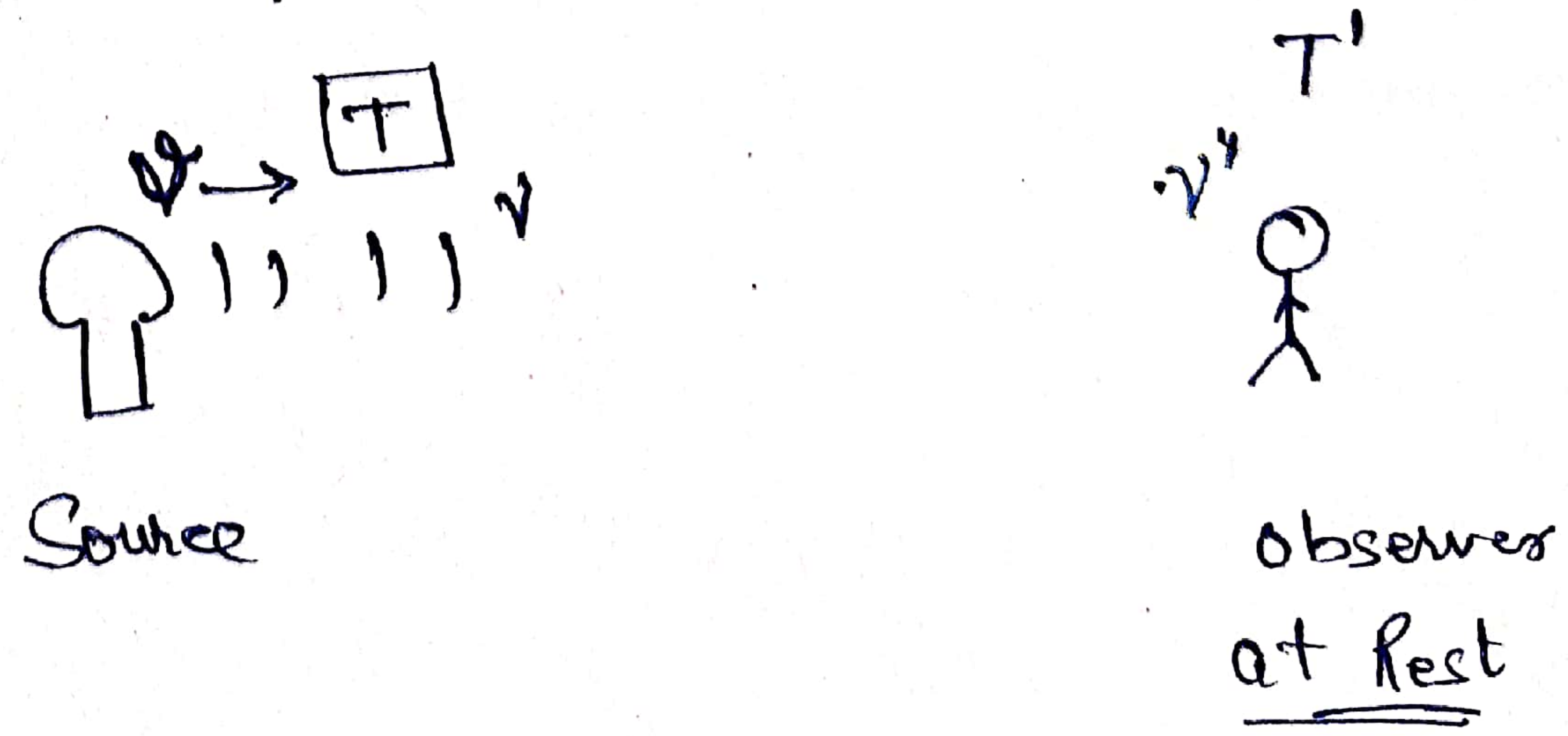
Speed very high → Time not same (in S 4 0)

Relative motion → Comparable to c

Source is moving with speed of v

EM freq → ν

Time period = T



$$T = \frac{1}{\nu}$$

For the observer →

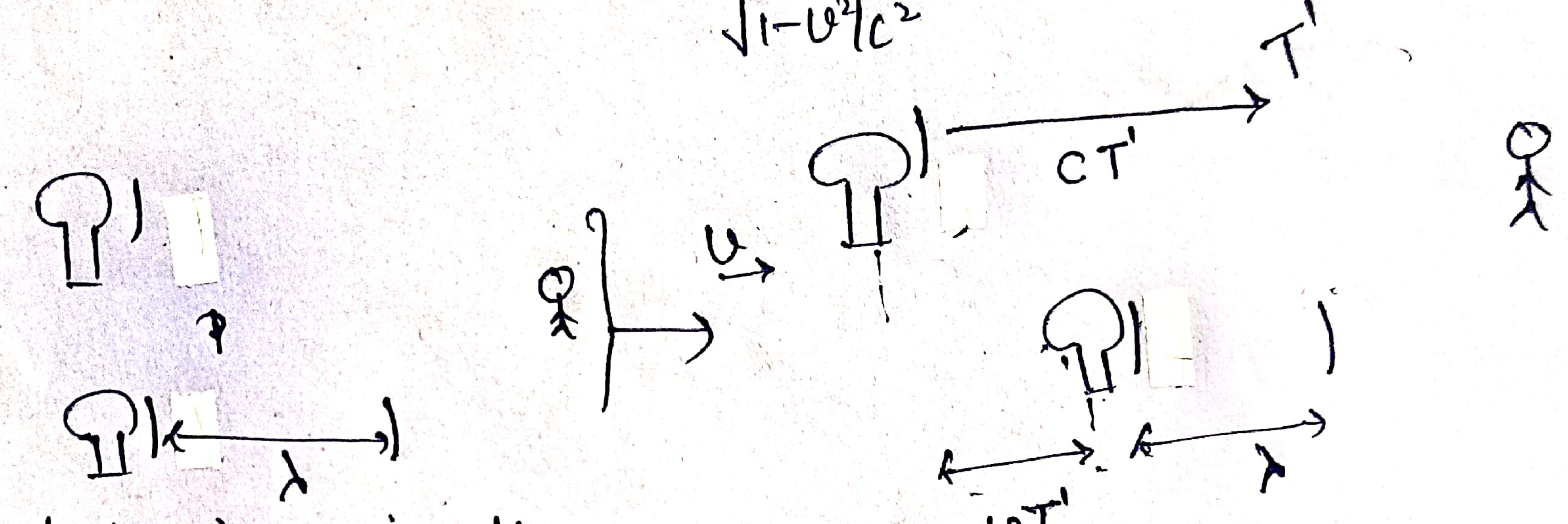
Time is not T →

observed frequency → ν'

Time → T' (d: Time dilation)

$$T' = \gamma T$$

$$T' = \frac{T}{\sqrt{1 - v^2/c^2}}$$



Source is moving v

$$\lambda = cT' - vT'$$

$$\lambda = (c - v)T'$$

$$\frac{c}{\nu'} = (c - v)T'$$

$$\nu' = \frac{c}{(c - v)T'}$$

$$\nu' = \frac{c}{(c - v)\gamma T}$$

Put $T = \frac{1}{\nu}$

$$\nu' = \frac{c\,\nu}{(c - v)\gamma}$$

put $\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$

$$\boxed{\nu' = \frac{c\,\nu}{(c - v)} \cdot \sqrt{1 - v^2/c^2}}$$

$$\nu' = \frac{c\,\nu\,\sqrt{c^2 - v^2}}{c\,(c - v)}$$

$$\nu' = \nu\sqrt{\frac{c + v}{c - v}}$$

$$\boxed{\nu' = \nu\sqrt{\frac{c + v}{c - v}}} = \boxed{\nu\sqrt{\frac{1 + v/c}{1 - v/c}}}$$

General formula

v	+ve	-ive
	toward	away

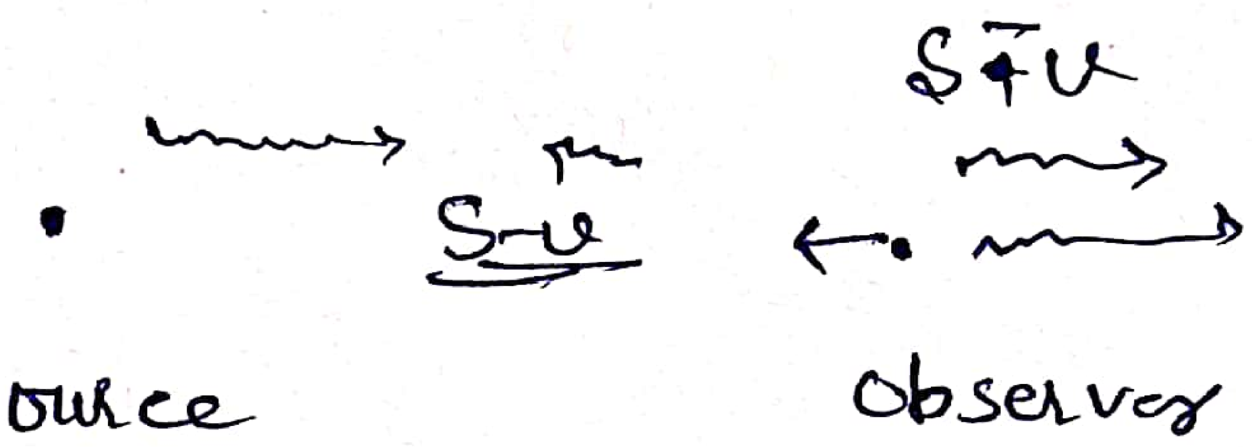
$$\nu = \nu_0 \frac{(1 + v/c)}{(1 - V/c)}$$

$\nu \rightarrow$ observed frequency

$\nu_0 \rightarrow$ actual source frequency

$v \rightarrow$ Speed of observer (toward + motion away −

$V \rightarrow$ Speed of source + toward + Reverse −



observer stationary $v = 0$ and source $V = 0$

$$\nu = \boxed{\nu = \nu_0}$$

ticks → per second. $t_0 = \frac{1}{\nu_0}$

elapses → Pass

$$t = t_0 / \sqrt{1 - v^2/c^2}$$

$$\nu = \frac{1}{t} = \frac{\sqrt{1 - v^2/c^2}}{t_0}$$

$$\boxed{\nu = \nu_0 \sqrt{1 - v^2/c^2}}$$

$$\nu = \frac{1}{t} = \frac{\sqrt{1 - v^2/c^2}}{t_0}$$

$$\nu = \nu_0 \sqrt{1 - v^2/c^2}$$

$$\boxed{\nu < \nu_0}$$